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# SUPER CONVERGENT SUM RULE FOR PION PHOTOPRODUCTION ON $\Lambda^o$ *

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ABSTRACT

A 'superconvergent' sum rule in the process of photoproduction of pion on $\Lambda$ hyperon is derived and studied within the framework of SU(3) symmetry. The sum rule is very well satisfied. The transition magnetic moment for the process $Y_1^* \rightarrow \Lambda\gamma$ is predicted to be $-2/3 \cdot \sqrt{2} \cdot \mu_2 \cdot (1.15)$.

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'Superconvergent' sum rules for certain strong interaction amplitudes have been recently derived by using the current algebra technique together with the assumption of unsubtracted dispersion relations [1] or on the basis of analiticity and appropriate high energy behavior of the amplitude [2]. Regge pole model has been frequently invoked in deriving such sum rules [3, 4].

In the present work we study a superconvergent sum rule in the photo-production of pion on $\Lambda$. The sum rules obtained for photo productions of pions on nucleons, discussed recently [4], are in fair agreement with the experimental data.

The invariant amplitude in the photoproduction process can be decomposed in terms of four invariant amplitudes [5], A, B, C, D. They are functions of the two invariants $\nu = - k\cdot(p_1 + p_2)/2M$ and $t = -(k-q)^2$ where $k$, $q$, $p_1$, $p_2$ are the four-momenta of the photon, meson, the initial baryon and the final baryon respectively. From the Regge pole theory, we assume the high energy behavior of each amplitude is determined by the leading Regge trajectory which can be exchanged in the t-channel. The recent Regge-pole analysis [6] of high energy scattering data suggests, if only $0^-$, $1^-$ and $2^-$ trajectories are assumed to be important in our case, that the invariant amplitude C behaves like $\nu^{\alpha(t)-2}$ for large $\nu$. Here $\alpha(t)$ refers to the leading trajectory, the $\omega$-trajectory in our case and for which $\alpha(0) < 1$ for $t \sim 0$. The amplitude C in odd under crossing symmetry i.e. $C^*(\nu, t) = - C(-\nu, t)$ and consequently leads to the non-trivial sum rule:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, C(\nu, t)\, d\nu = 2 \int^{\infty} \mathrm{Im}\, C(\nu, t)\, d\nu = 0 \quad ; \text{fixed } t.$$

The pole term contribution to the integral due to $\Sigma$ intermediate state is readily evaluated while the continuum contribution may be approximated using isobaric model retaining only $Y_1^*$ (1385) contribution.[7, 8] The crossing symmetry relation implies that we need consider only the direct uncrossed graphs in the s-channel ($\gamma + \Lambda \longrightarrow \Lambda + \pi$). In fact, due to crossing relation the contributions, at fixed t, to the above integral coming from the left-hand pole and the left-hand cut, corresponding to the crossed u-channel contributions to the s-channel is equal to the contributions from the right-hand pole and the right hand cut.

The various coupings needed for our calculation are:

$$\Sigma \Lambda \gamma : \left(\frac{e}{2M_N}\right) \mu_{\Sigma\Lambda} \bar{\psi}_{\Sigma} \frac{\sigma^{\mu\nu}}{2} \psi_{\Lambda} \; F_{\mu\nu} \; + \text{h.c.}$$

$$\Sigma \Lambda \Pi : \; g' \, \bar{\psi}_{\Sigma} \; \gamma^5 \; \gamma_{\alpha} \, \psi_{\Lambda} \, \varphi_{\alpha} \; + \text{h.c.}$$

g' may be obtained from, say, SU(3) D-type coupling of $\bar{B}BP$.

Following Gourdan and Salin [8] we define the following couplings:

$$Y_1^* \Lambda \Pi^{\circ} : \; i\left(\frac{\lambda_1}{\mu}\right) \left[\bar{\psi}\, \psi^{\rho}\, \partial_{\rho}\, \varphi - \partial_{\rho}\, \varphi^{*}\, \bar{\psi}^{\rho}\, \psi\right]$$

where $\psi$, $\psi^{\rho}$, $\varphi$ represent the fields corresponding to $\Lambda$, $Y_1^*$ and pion respectively. $\lambda_1$ is given by:

$$\left(\frac{\lambda_1}{\mu}\right)^2 = \frac{12\pi}{(E+M)} \frac{M\Gamma}{q^3}$$

where E is the nucleon c.m. energy at the resonance, q is the c.m. momentum at the resonance energy, $\Gamma$ the width of the decay and M is the mass of .

$$Y_1^* \Lambda \gamma: \quad \frac{ie\, C_3}{\mu} \left[\bar{\psi}^{\mu} \gamma^{\nu} \gamma^5 \psi + \bar{\psi} \gamma^5 \gamma^{\nu} \psi^{\mu}\right] F_{\mu\nu}$$

$$+ \frac{e\, C_4}{\mu^2} \left[\bar{\psi}^{\mu} \gamma^5 \partial^{\nu} \psi + \partial^{\nu} \bar{\psi} \gamma^5 \psi^{\mu}\right] F_{\mu\nu}$$

where the constants $C_3$ and $C_4$ determine the magnetic dipole and electric quadrupole transition moments for the electromagnetic transition $Y_1^* \rightarrow \Lambda + \gamma$. In fact, we may define (q) the magnetic dipole transition moment as:

$$\mu^* = \frac{1}{2} \left\{ \frac{M^{*2} - M^2}{\mu^2} C_4 + \frac{M^* + 2M + E_1}{\mu} C_3 \right\}$$

where $E_1$ is the c.m. energy of $\Lambda$ in $\gamma\Lambda$ c.m. frame. In the static limit we replace $E_1$ by M. The Glebsch-Gordan coefficient factor $\sqrt{\frac{2}{3}}$ appearing in the case of $N^*_{3/2}$ does not appear in the present case since the $Y_1^*$ iso-spin is unity and the iso-spin of $\Lambda$ is zero.

The Feynman graphs corresponding to $\Sigma$ and $Y_1^*$ are easily calculated and the amplitude C projected out by trace method. In fact if we write the total invariant amplitude as $\bar{u}(p_2)\, O\, u(p_1)$ then the amplitude C is simply proportional to $Tr(i\gamma\cdot[p_1 + p_2]\gamma_5\, O)$.

For $\Sigma$ intermediate state the contribution to C is:

$$g' \mu_{\Sigma\Lambda} \left( \frac{e}{2M_N} \right) \frac{1}{(W^2 - M^2 + i\epsilon)}$$

Consequently, the contribution to Im C is:

$$- g' \mu_{\Sigma\Lambda} \left( \frac{e}{2M_N} \right) \pi \delta (\rho - M_{\Sigma}^2)$$

where $W^2 = s$ is the square of the c.m. energy in the s-channel $(\gamma + \Lambda \longrightarrow \Lambda + \pi)$.

The contribution due to the decuplet $Y_1^*$ intermediate state is found to be:

$$\frac{e\lambda_1 C_3}{\mu^2} \frac{1}{(W^2 - M^{*2} + i\gamma')} \left[ (W + M) - \frac{1}{6W} \left\{ (E_2 + M)(3W + M) + 3(W+M)q_o \right\} \right]$$

$$+ \frac{e\lambda_1 C_4}{\mu^3} \frac{1}{(W^2 - M^{*2} + i\gamma')} \left[ \frac{W^2 - M^2}{2} - \frac{1}{12 W} \left\{ 2W(W-M)(E_2 + M) \right.\right.$$

$$\left.\left. + 3 q_o(W^2 - M^2) + \frac{1}{2} |\vec{k}| |\vec{q}| \text{ Cos } \theta \right\} \right]$$

where in the c.m. frame we have

$$2WE_1 = W^2 + M^2$$

$$2WE_2 = W^2 + M^2 - \mu^2$$

$$2Wq_o = W^2 - M^2 + \mu^2$$

$$2|\vec{k}| |\vec{q}| \text{ Cos }\theta = (t - \mu^2) + \frac{1}{2W^2} (W^2 - M^2)(W^2 - M^2 + \mu^2)$$

Contribution to the ImC is:

$$\left\{ \frac{e\lambda_1 C_3}{\mu^2} [\ ] + \frac{e\lambda_1 C_4}{\mu^3} [\ ] \right\} \left( \frac{-\gamma'}{(W^2 - M^{*2})^2 + \gamma'^2} \right)$$

We may approximate the quantities inside $\{\ \}$ by putting $W^2 = M^{*2}$, $M^*$ being the mass of $Y_1^*$, since the factor depending on the width $\gamma'$ is like $\delta(N^2 - M^{*2})$ in the limit of narrow width. Thus we obtain for the Im C:

$$- \pi g' \mu_{\Sigma\Lambda}\left(\frac{e}{2M_N}\right) \delta(\rho - M_\Sigma^2)$$
$$+ \frac{e\lambda_1}{\mu}\left(6.28\, C_3 + \left[17.72 + \frac{t}{4\mu^2}\right] C_4\right) \quad \frac{-\gamma'}{(W^2 - M^{*2})^2 + \gamma'^2}$$

Substituting in the sum rule

$$\int_{\rho = M^2}^{\infty} ds\ \text{Im}\ C(s, t) = 0 \qquad ;\ \text{fixed } t.$$

we obtain for, say, $\gamma' \sim 140$ MeV or so:

$$g'\mu_{\Sigma\Lambda} = - \frac{2M_N \lambda_1}{\mu} \ (6.28\, C_3 + 17.72\, C_4)$$

where we have put $t \sim 0$.

Assuming that only the $M_{1+}$ pole is mainly responsible for the photoproduction, as is also verified experimently in the case of pion photoproduction [8], we have:

$$E_{1+} = 0 \qquad \text{giving} \qquad C_4 = -\left(\frac{\mu}{2W}\right) C_3$$

and the sum rule relation reduces to:

$$g' \mu_{\Sigma\Lambda} = - 10.84 \ \frac{M_N}{\mu} \ C_3 \lambda_1$$

Assuming for the transition magnetic moment $\mu_{\Sigma\Lambda}$ and the coupling $g'$ their SU(3) values:

$$\mu_{\Sigma\Lambda} = - (\sqrt{3}/2)\, \mu_n$$

$$g' = \frac{2}{\sqrt{3}}(1-\alpha)g$$

where $g^2/4 = 15$ and $(1-\alpha)/\alpha$ is the D/F ratio for the BBP vertex, we obtain

$$g(1-\alpha) = -39.44\, C_3\, \lambda_1$$

$\lambda_1^2$ can be calculated from the known width of the decays $Y_1^{*\pm} \rightarrow \Lambda + \pi^{\pm}$ and using the charge independence. We find $\lambda_1 = \mp 1.16$ thereby obtaining

$$C_3 = \pm\, 0.3\,(1-\alpha)$$

The magnetic dipole moment of $Y_1^* \Lambda^o$ transition is given by ($C_4 = -\frac{\mu}{2M^*} C_3$).

For $\alpha \sim 0.4$, the SU(6) value we find $C_3 = -0.18$ and $\mu^* = -\frac{2}{3}\sqrt{2}\,\mu_p(1.15)$ where $\mu_p = 2.79$ is the total proton magnetic moment. The + sign is taken for $\lambda_1$ as suggested by [10] SU(6). Within the framework of SU(3) symmetry we have the relation [11]

$$\mu^*(N^{*+}_{3/2}\, p\gamma) = \mu^*(N^{*o}_{3/2}\, n\gamma) = -\frac{2}{\sqrt{3}}\mu^*(Y_1^*\Lambda\gamma)$$

The result obtained by us thus implies $\mu^*(N^{*+}_{3/2}\, p\gamma) = +\frac{2}{3}\sqrt{2}\,\mu_p(1.30)$ which is to be compared with the experimental value $+\frac{2}{3}\sqrt{2}\,\mu_p$ $(1.28 \pm 0.02)$.

Thus we conclude that the sum rule obtained above is very well satisfied within the framework of SU(3) symmetry and the transition magnetic moment for $Y_1^* \longrightarrow \Lambda^o\gamma$ turns out to be $-\frac{2}{3}\sqrt{2}\,\mu_p(1.15)$.

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