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PULSAR SLOW-DOWN EPOCHS

by

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Abstract:

The relative importance of magnetospheric currents and low frequency waves for pulsar braking is assessed and a model is developed which tries to account for the available pulsar timing data under the unifying aspect that all pulsars have equal masses and magnetic moments and are born as rapid rotators. Four epochs of slow-down are distinguished which are dominated by different braking mechanisms. According to the model no direct relationship exists between "slow-down age" and true age of a pulsar and leads to a pulsar birth-rate of one event per hundred years.

Based on theoretical arguments about the progenitors of pulsars there exists the possibility that all neutron stars have essentially the same mass[1) 2)] M and the same magnetic moment[3) 4)] $\vec{M}$. The best direct determinations[5) 6)] support this view and give $M \approx 10^{33,5}$g (the Chandrasekhar mass) and $\vec{M} \approx 10^{30,5}$ Gauss cm^3 and theoretically inferred values for accreting binary systems[7) 1)] show a surprisingly small scatter. Can this apparent uniformity for the binary pulsars be reconciled with the timing data for (single) radio pulsars, of which many may also have been binaries for some time? After the identification of radio pulsars with rotating, magnetized neutron stars[8)] and the proof that they must be surrounded by a magnetosphere[9) 10)] independent of the work function of the neutron stars surface[11) 12)] progress in understanding the long-scale aspects of the magnetosphere, which determines the braking of the neutron star's rotation, has been slow[13)]. The theoretical analysis of pulsar braking is hampered by two facts: no self-consistent solution for a pulsar magnetosphere has been found and the pulsar timing data seem to reveal more about the neutron star's interior than about its magnetosphere. The profuse wealth of radio observations can at best be used as a diagnostic [13)] for the slow-down process. There is nevertheless no lack of theoretical models trying to explain the observational data and in most theories it is assumed that the neutron star is slowed-down by the combined action of a plasma-current-torque and a vacuum-wave-torque. This leads to the well-known result that there is a deficiency in "old" pulsars (as measured by their "Slow-down age" $^P/\dot{P}$) and the inferred magnetic moments vary by two orders of magnitude. Worse still is the derived pulsar birth rate[2)] of one event per ten years if the half-life of a pulsar is 10^6 years as follows from the standard slow-down theory. These facts are the main excuse to present a new model for pulsar slow-down and it seems appropriate to list the assumptions on which the model is based: 1. "young" pulsars produce so much plasma by means of "sparking"[11) 12)] that <u>within</u> the plasma no low-frequency wave can propagate[15) 16)]. 2. as pulsars "grow older" sparking becomes less effectives so that eventually low-frequency waves can be emitted <u>within</u> the plas ma. 3. out to the velocity-of-light-cylinder (i.e. that part of the magnetosphere which, were it to corotate rigidly, would rotate at the speed of light) the Goldreich-Julian[9] model as extended to the oblique rotator by Mestel[10] describes the long-term aspects of the magnetosphere. Minor modifications such as a current-regulating net charge and discharges due to radiation friction will be worked later into the model. 4. Considerable perpendicularity between magnetic moment and the spin of the neutron star occurs during the first epoch, where the torque is dominated by currents in the plasma. The present model accounts for this perpendicularity if the star can be treated as a sphere so that free nutation is not possible[17) 18)]. However any other (internal) mechanism which leads to considerable perpendicularity will lead to the same consequences (cf. ref. 18 and the further references quoted therein). The Goldreich-Julian model of the pulsar magnetosphere predicts an excess charge-density in the magnetosphere

$$q \approx - \frac{\vec{\Omega}.\vec{B}}{2\pi c} \tag{1}$$

and an average current

$$\vec{j} \approx qc\vec{b}_{.} \tag{2}$$

along the open field lines which leave away from the surface area ΔF centered on the magnetic poles ("polar caps"). Here $\vec{\Omega}$ is the spin angular velocity, $\vec{B}$ the magnetic field, c the velocity of light and $\vec{b}_0$ a unit vector in the direction of $\vec{B}$. For the star not to charge up indefinitely there must be a back-current which flows along magnetic field lines further away form the centre of the polar cap. It will be regulated by a net charge as discussed below. To close the current charges must flow within the neutron star across the magnetic field lines and it is this current which breaks the neutron star's rotation. In the Goldreich-Julian model it is assumed that energy and angular momentum are dissipated beyond the velocity-of-light-cylinder. Within the velocity-of-light-cylinder the charges move along the magnetic field lines like beads on a wire and by their current provide thus a "magnetic spring" between the neutron star's surface and the matter beyond the velocity-of-light-cylinder. Its torque $\vec{T}$ is given by

$$\vec{T} = - \frac{1}{4\pi r} \int (\vec{r}.\vec{B})\vec{r} \times \vec{B} dF \qquad (3)$$

Where $\vec{r}$ is the radius vector counted from the centre of the star and the integral is over a sphere of radius r. The current of equ. (2) leads to a counteraligment torque[18] between $\vec{\Omega}$ and $\vec{M}$, in contradistinction to the torque exerted by low-frequency waves propagating in vacuo, which (if not impeded by nutation[17]) leads to aligment[19] [20]. The counteraligment torque is easily understood if one notes that a current flowing through a magnetized sphere will set the sphere into rotation about magnetic dipole axis $\vec{M}$ and the current of equ. (2) is so directed (lenz'rule) that is reduces the rotation about the original axis. Both in the plasma and in the vacuum case the star acts such as to minimise the applied torque and stores some rotational energy into rotation about a new rotation axis. In the dipole approximation the polar cap surface are ΔF is given by[9] [11] $\Delta F \approx 2\pi R^2 (\frac{\Omega R}{c})$ where $R \approx 10^6$ cm is the radius of the star and in a coordinate system centered on the magnetic dipole-axis we find for the toroidal component of the magnetic field

$$B_\phi \approx - \frac{\vec{\Omega}\vec{B}}{2\pi c} \int \frac{dF}{R \sin\theta} \qquad (4)$$

which leads by means of equ. (3) to

$$\vec{T}_{p\ell} = -\alpha \frac{\Omega^2}{c^3} (\vec{\Omega}.\vec{M})\vec{M} \qquad (5)$$

where $\vec{M} = R^3\vec{B}$ is the magnetic dipole moment and $\alpha \approx 1$. From the corotating part of the magnetosphere we obtain an induced magnetic field parallel to the rotation axis which leads to an extra torque (by means of the magnetic dipole moment $\vec{M}$) on the star

$$\vec{T}'_{p\ell} = \frac{\gamma}{Rc^2}(\vec{\Omega}.\vec{M}) \; \vec{\Omega} \times \vec{M} \qquad (6)$$

where $\gamma \approx 1$. Equs (5) and (6) may be compared to the vacuum wave torque[17) 19) 20)]

$$\vec{T}_w = -\beta \frac{\Omega^2}{c^3}(\vec{M} \times \vec{\Omega}) \times \vec{M} + \frac{1}{Rc^2}(\vec{M} \cdot \vec{\Omega}) \vec{\Omega} \times \vec{M} \tag{7}$$

with $\beta = {}^2/3$. It has been common to assume that both plasma and low-frequency waves will (somehow) contribute (more or less equally) to the slow-down torque, which leads to the well-known large scatter in inferred magnetic dipole moments. This assumption will be shown now to be quite wrong. If low-frequency waves cannot propagate within the plasma they cannot exist outside either. Apart from a transition period where the plasma may just allow for low frequency wave propagation (the duration of which is difficult to estimate as it depends on the sparking mechanism) a pulsar is slowed-down exclusively by either the plasma-current-torque equ. (5) or the vacuum-torque equ. (7).

The most favourable conditions for low-frequency wave-emission obtain for the orthogonal rotator and we assume that the plasma which flows out of the velocity of light cylinder fills the space about the equatorial plane, consequently the waves are emitted into a plasma deadzone centered on the rotations axis. Let us assume that the two zones are separated by a cone of half-angle ψ (counted from the roation axis) and as a first approximation[16)] that the plasma is infinitely well conducting. This problem can be solved exactly. The solution of the vector-Helmholtz-equation may be taken from Morse and Feshbach[23)]. One finds that the TEM-mode dominates and the dominant radiation mode is given by the lowest n for which

$$\frac{n+1}{n} P^1_{n-1}(\cos\psi) - \frac{n}{n+1} P^1_{n+1}(\cos\psi) = 0 \tag{8}$$

$$P^1_n(\cos\psi) = \frac{n(n+1)}{2} \sin\psi F(1-n,\ 2+n|2|\ \frac{1-\cos\psi}{2}) \tag{9}$$

F in equ. (9) is the hypergeometric function which for noninteger n is regular in the upper hemisphere where equs. (8) and (9) hold. If we let the conducting cone shrink to the equatorial plane one obtains the well-known Deutsch solution[24] with n = 1. For a thin plasma sheet ($\psi = {}^{\pi}/2 - \varepsilon$) one finds approximately

$$12 = (1-\cos\psi)(n^2(n+2)(n+3)-(1-n^2)(1+n)(2-n)) \qquad (10)$$

which shows that n is larger than one. The radiated energy rate is

$$\dot{E} \approx -\frac{2}{3cR^2}(\vec{M} \times \vec{\Omega})^2 \left(\frac{\Omega R}{c}\right)^{2n} \qquad (11)$$

For a finite thickness of the plasma sheet the emission of low-frequency waves is so strongly reduced ($n = 2$ for $\psi = {}^{\pi}/4$) that the wave pressure cannot balance the plasma pressure at the boundary and the plasma fills the whole space. As an aside we note however that if the plasma is asymmetrically distributed in the two hemispheres, such that one cone has $\psi > {}^{\pi}/2$ (plasma swept backward e.g. by the pulsar's proper motion?[25]) radiation emission is enhanced and such a state may be called superradiant. A young pulsar will therefore be slowed-down exclusively by the plasma current, an old one by the wave-torque and in the transition epoch we may have

$$\dot{\vec{J}} = I\dot{\vec{\Omega}} = (\beta-\alpha)\,\frac{\Omega^2}{c^3}(\vec{M}\vec{\Omega})\vec{M} - \beta\,\frac{\Omega^2}{c^3}\,M^2\vec{\Omega} + \gamma\,\frac{\vec{\Omega}.\vec{M}}{Rc^2}\,\vec{\Omega} \times \vec{M} \qquad (12)$$

where $\vec{J}$ is the angular momentum and I the moment of inertia of the neutron star, $I \approx 10^{45}\ \mathrm{gcm}^2$. Together with the "equation of motion" for the dipole moment $\vec{M}$, which is frozen into the star

$$\dot{\vec{M}} = \vec{\Omega} \times \vec{M} \qquad (13)$$

one obtains easily the evolution of the slow-down. With the help of the first integral

$$\frac{(\Omega^2\sin^2\chi)^{\alpha}}{(\Omega^2\cos^2\chi)^{\beta}} = \mathrm{const} \qquad (14)$$

where χ is the angle between $\vec{\Omega}$ and $\vec{M}$ we obtain for young pulsars, for which $\beta = 0$,

$$\sin\chi = \sin\chi_i (\frac{\Omega_i}{\Omega}) \tag{15}$$

$$\Omega = \Omega_i (1+\mathrm{ctg}^2\chi_i (1-e^{-\tau})^{-1/2} \tag{16}$$

which reads for small times

$$\Omega = \Omega_i (1- \frac{\tau}{2} \mathrm{ctg}^2\chi_i + \frac{\tau^2}{8} (2\mathrm{ctg}^2\chi_i + 3\mathrm{ctg}^4\chi_i)) \tag{17}$$

where $\tau = 2\alpha\Omega_i^2 \sin^2\chi_i \frac{M^2}{Ic^3} \tau = {}^{\tau}/\tau_e$ is a dimensionless parameter which measures the observer's time t in units of the e-folding time τ_e of the model. The index is refers to initial values. Before we turn to a discussion of equ. (16), which (apart from the demonstration that plasma and vacuum waves cannot coexist) is our main result, let us discuss briefly one further important parameter for pulsar timing observations. The so called braking index $N = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2}$ is given in our model by

$$N = 3 + 2 \frac{(\beta-\alpha)^2 \cos^2\chi \sin^2\chi}{(\alpha \cos^2\chi + \beta \sin^2\chi)^2} \tag{18}$$

and is never smaller than three due to torque minimisation. Observationally N is known only reliably for the Crab pulsar[26)27)] where $N \approx 2{,}5$. Rewriting the energy balance equation in the form

$$\frac{1}{2}(I\Omega^2)^{\bullet} = - \frac{\alpha}{cR^2} (\vec{\Omega} \vec{M}) (\frac{\Delta F}{F})^2 \tag{19}$$

where $F = 4\pi R^2$ is the surface area of the neutron star, we see that a braking index smaller than three may be explained if the pulsar's crust is shrinking[21) 22) 28)] at a rather large rate, or by a slightly larger polar cap $\frac{\Delta F}{F} = (\frac{\Omega R}{c})^{2/3}$. In fact in some theories[29)] the pulse width ΔP and the period P are related as $({}^{\Delta P}/P)^2 = {}^{\Delta F}/F)$ and the observations of the Crab pulsar, where ${}^{\Delta P}/P \sim {}^1/5$ is rather large[29)], would fit better with ${}^{\Delta F}/F \simeq ({}^{\Omega R}/c)^{2/3}$ leading to a braking index $N = 3 - {}^2/3 + 2\,\mathrm{tg}^2\chi$.

Let us show now that the model is flexible enough to account for the available timing data under the severe restriction that all pulsars have the same moment of inertia $I = 10^{45}$ gcm^2 and the same magnetic moment $M = 10^{30,5}$ Gauss cm^3. To obtain t_e from the observations we identify those pulsars with anomalously low period derivative with the stars in our model which pass through the end of the first epoch. We have from the observations $\Omega \approx 2\pi sec^{-1}$ which gives $\Omega_i . \sin\chi_i = 2\pi\ sec^{-1}$ so that

$$t_e \approx 10^6 \text{ years } I_{45} M_{30,5} \tag{20}$$

Observationally the two most extreme cases are the Crab pulsar and the binary pulsar. Equ. (20) would lead for them to $\sin \chi_i = 10^{-1,5}$ and $\cos \chi_i = 10^{-1,5}$ respectively if we assume that both are young objects. To explains the binary pulsar in this way one needs a nearly orthogonal rotator and one may worry if equ. (12) is still valid for this case. It requires $\beta < 10^{-4}$ and $\cos^2\chi < 10^{-3}$. For the binary pulsar $(^{\Omega R}/c)^2 < 10^{-5}$, which according to the previous analysis guarantees that $\beta < 10^{-4}$ and inspection of the current as given by equ. (2) shows that it can be closed along the magnetic field lines through the star so that it does not lead to a torque. The braking is then no longer effeted by the current of equ. (2) but comes about through secondary energy losses such as sparking. Taking Ruderman's estimate[12)] of that energy for the (faster) Crab pulsar of 10^{33}-10^{34} ergs sec^{-1} we see that this would just lead to the observed braking of the binary pulsar. The first epoch, wich lasts some 10^6 years accounts for roughly one half of the pulsars under the assumption that all are born as fast rotators. The other half can be explained in the penultimate epoch of pulsar slow-down, where vacuum waves can be emitted in the presence of plasma so that the period derivative goes back to its "normal" value.

Note that in the present model the "slow-down age" is not related to the true age, only the period itself is a crude measure of it. The mean active life as determined by Ohmic dispation can exceed easily 10^7 years[18], which brings down the pulsar birth rate by a factor of ten, in comfortable agreement with the more conservative estimates of super-nova rates and the lack of discovered neutron stars at their centers[36].

Before we discuss the final epoch let us discuss some subtle points of the present model. We have so far only assumed that the current of equ. (2) flows on the average without demonstrating how it comes about. Of course a rigourous demonstration requires a self-consistent solution of the magnetosphere problem, so only the following qualitative argument can be given. According to the Goldreich-Julian model particles cannot stay within the velocity of light cylinder for the same reason that they cannot stay within the star: large electric fields would pull them out. The effect is such that the charge with the correct sign as given by equ. (1) will be pulled <u>out</u>, charges with the opposite sign however are pulled <u>in</u> on the same field line. This shows that a pulsar must have a net charge[30] Q to regulate the plasma out-flow such that the star does not charge up indefinitely. Some of the charge will be distributed over the polar cap ΔF and most of it over the boundary of the corotating magnetosphere and as it must be able to influence the dynamics of the plasma at the velocity-of-light cylinder it must be of the order of

$$Q \approx \frac{\vec{\Omega}\cdot\vec{M}}{c} \tag{21}$$

Such a charge reintroduces what the Goldreich-Julian model tried to avoid: large electric fields, so that we have essentially shifted the whole problem from the surface of the neutron star to its velocity-of-light-cylinder, sufficiently far away however that the star does not get heated too much[1) 36)]. Note that the net charge as given by equ. (21) will not give rise to a back-current from the interstellar matter to the pulsar during its "active life" as the pulsar is well shielded by the el.mag.fields of the magnetosphere or the vacuum waves which both fall off like r^{-1} whereas the monopole field falls off like r^{-2} so that the force balance is in fact at the velocity of light cylinder. In the penultimate slow-down era, which is dominated by low-frequency waves this charge and the corotating (quadrupole) charge of the magnetosphere will also radiate and this leads to a friction force on the magnetic field lines with non-vanishing curl. To compensate for this, the particles must drift across magnetic field lines giving rise to a net current out of the corotation zone. For the quadrupole radiation from the corotating magnetosphere we get for the time-scale of the ensuing discharges some 10^6 pulsar periods and a much shorter time scale for the dipole radiation due to the charge given by equ. (21). These discharges may be related to the nulling phenomenon and may give rise to slow-down noise[27) 28)] but not to any directly observable speed-ups as the inertia involved is too small. The present model does not explain why pulsars turn off unless sparking ceases to be regular enough to allow an observer to detect the object as a pulsar, but it appears that even accretion may influence the final era[31)] especially if the pulsar has become an aligned rotator by then. An attractive explanation is obtained if one combines the pulsar extinction hypothesis[32) 33)] with the decay of the magnetic dipole moment[18)] which regulates the physics at the velocity-of-light-cylinder. The observed cut-off period P $\approx$ 4 sec would then not be mainly a consequence of plasma inertia but rather reflect the time-scale for Ohmic dissipation in the pulsar's crust which may vary considerably from pulsar to pulsar depending on its thermal history at birth. According to the present model such neutron stars will slow-down in the final era by quadrupole radiation (or plasma currents) on a time scale exceeding the age of the universe and only if they traverse dense interstellar matter may they be slowed-down more effectively by accretion[31)]. The γ - ray transient[34)] with a period of 8 sec could in fact be such an old pulsar, whereas the γ-ray source CG 195,5 + 4,5, if its periodicity of 59.35 sec should be confirmed[35)], should rather not be identified with an old pulsar according to the present model, which predicts ultimate periods around 10 seconds for dead pulsars, instead of

60 seconds as deduced by Michel[32] under the assumption that the dipole field does not decay.

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