ISSN 0029-3865

CBPF-NF-012/86

SUPERGRAVITY*

by

Prem P. Srivastava

Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Lecture delivered at the "Simposio de Relatividade Geral e Gravitação", CBPF, Rio de Janeiro, Dec. 1985 (Based on a chapter of the forthcoming book: Supersymmetry, Superfield and Supergravitation. An Introduction, Adam-Hilger, England).

CBPF-NF-012/86

ABSTRACT

A comprehensive introduction to the theory of supergravity is given and the super-Higgs effect illustrated by considering several examples of the Kähler potential occuring in the general coupling to the Yang-Mills system.

Key-words: Supergravity; Supersymmetry; Gravity; Relativity; Gauge Theories.

1 INTRODUCTION

Rigorous rigid supersymmetry, e.g., exact supersymmetry of the Lagrangian and the vacuum, implies degeneracy among bosonic and fermionic energy levels. Applying a s.s. generator to any bosonic (fermionic) state which is not annihilated by it we create a fermionic (bosonic) state with the same energy and momentum. All the known elementary particles should have consequently superpartners with the same mass. No such mass degeneracy is observed in nature. We list below some of the superpartners required when we attempt to build a supersymmetric unified theory of fundamental interactions[1]

Particle	Spin	Sparticle	Spin
Quark q_L, q_R	1/2,1/2	Sqark $\tilde{q}_L, \tilde{q}_R$	0,0
Lepton l_L, l_R	1/2,1/2	Slepton $\tilde{l}_L, \tilde{l}_R$	0,0
Photon γ	1	Photino $\tilde{\gamma}$	1/2
Gluon g	1	Gluino $\tilde{g}$	1/2
$W^{+,-}$	1	W-ino $\tilde{W}^{+,-}$	1/2
Z^0	1	Z-ino $\tilde{Z}^0$	1/2
Higgs H	0	Shiggs $\tilde{H}$	1/2

Table 1: Superpartners of some particles required for a superunified theory of fundamental interactions.

If supersymmetry is to be relevant for the physical world it must be broken spontaneously or softly so that some of the superpartners may be made massive to be in agreement with the present experiments[a], for example, $m_{\tilde{q}}, m_{\tilde{l}}, m_{\tilde{W}}, m_{\tilde{H}} \doteq 0$ (15-20)GeV.

We also know that when the global supersymmetry is realized in the spontaneously broken mode in which the s.s. generators do not annihilate the vacuum there appears in the theory a massless spin-1/2 Majorana fermion-'goldstino'-field. It is not possible to identify, for example, the 'goldstino' field with the electron neutrino because it would satisfy low energy theorems which contradict the observed properties of the neutrino spectrum. The problem of the apparent non-existence of the 'goldstino' particle may be overcome by lifting the global supersymmetry to a local one where the transformation parameters become space-time dependent and a spin-3/2 compensating gauge field ψ_μ has to be added in the theory. The goldstino field in this context assumes the role of a gauge degree of freedom and a gauge can be chosen which eliminates it entirely from the theory while the gauge field ψ_μ itself become massive[2] - Super-Higg's Effect - analogous to the well known standard Higg's effect of ordinary gauge theories.

Among several other strong motivations for studying the local supersymmetry we may mention the problem of the observed smallness of the Cosmological constant Λ which implies in our universe a very small value for the vacuum energy density $V_o \simeq \Lambda \lesssim 10^{-120} M^4 \simeq (3x10^{-12} GeV)^4$ where $M^{-1} = (8\pi G_N)^{1/2}$. In the case of global supersymmetry the vacuum energy is the order parameter for broken supersymmetry and it is positive definite. The vacuum energy density gets only positive contributions proportional to the supersymmetry breaking scale excluding the possibility of cancellations to obtain small value if other constraints of particle phenomenology are also to be taken care of. On the other hand local supersymmetry (also called Supergravity theory) brings in necessarily the gravitation in the theory and the vacuum energy has the possibility to become positive, negative or vanishing. This offers a hope for adjusting at the classical level a very small value for the cosmological constant which may not be spoiled by quantum corrections because of the residual symmetries in the theory. We will mention in Sec. 3 such a model which seems to follow also from the recent investigations on superstring models in higher space-time dimensions. The supersymmetry also offers some hope for resolving certain other unsolved problems in the field of Cosmology as well.[3]

2 PURE N=1 SUPERGRAVITY

2.1 Local supersymmetry

When we promote a rigid (global) symmetry with constant parameters ϵ to a local symmetry by letting† $\epsilon \to \epsilon(x)$ the kinetic terms of the action are no longer invariant and we find

$$\delta I = \int d^4x \; j_N^m \; \partial_m \epsilon \tag{1}$$

where $j_N^m(x)$ is an on-shell conserved Noether's current of the global symmetry. In order to restore the symmetry we are required to introduce in the theory additional compensating gauge field. We may cancel the variation (1) by adding, apart from a kinetic term for the gauge field, a new interaction term (minimal coup-

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†For infinitesimal gauge variations $\delta\phi(x) \simeq \phi'(x) - \phi(x)$ we find on the mass-shell $\delta L \stackrel{\circ}{=} \partial_\ell[\delta\phi \frac{\partial L}{\partial(\partial_\ell \phi)}]$. If the action is invariant under the variations with constant parameters ϵ, viz, $\delta L = \epsilon \partial_\ell \Lambda^\ell(x)$ the conserved Noether current is given by $\epsilon j_N^\ell = \delta\phi \frac{\partial L}{\partial(\partial_\ell \phi)} - \epsilon \Lambda^\ell$. When this global symmetry becomes local, e.g., $\epsilon \to \epsilon(x)$ we find $\delta L \stackrel{\circ}{=} \partial_\ell[\epsilon(x) j_N^\ell + \Lambda^\ell] = \partial_\ell(\epsilon\Lambda^\ell) + j_N^\ell \partial_\ell \epsilon(x) + \ldots$ See for example, P.P. Srivastava, Nucl. Phys. B64(1973)499; Rev. Bras. Fis. 3(1973)577.

ling) to the action

$$I' = -\kappa\int d^4x\ j_N^m A_m \qquad , \qquad \delta A_m = \frac{1}{\kappa}\,\partial_m \epsilon + \ldots \qquad (2)$$

where κ is a constant and the transformation law of the gauge field contains the derivative of the symmetry parameter. But a new term will arise from δj_N^m which again has to be compensated by adding more terms to the action and possibly to the earlier transformation rules under the local symmetry of the fields involved as well. Working step by step the so called Noether procedure (which becomes an iterative procedure in κ) it may result in a locally symmetric action after a finite number of steps.

The procedure may be followed for supersymmetric theories as well. However, their transformation laws at the global level already contain the derivatives of the component fields. We assume that at the local level only the gauge fields transform with terms that are proportional to $\partial_m\epsilon$ while the other fields contain $\epsilon(x)$ but not $\partial_m\epsilon(x)$, e.g., $\delta\psi = -\sqrt{2}[i(\sigma_m\bar{\epsilon}(x))\partial^m A + F(x)\epsilon]$. The spinor parameter carries dimension -1/2 and it follows that if we require dimension 3/2 for the vector-spinor fermionic gauge field indicated by ψ_m the parameter κ will carry dimension $[\kappa] = -1, \delta\psi_m(x) = \frac{1}{\kappa}\ \partial_m\epsilon(x) + \ldots$ The necessity of the constant κ with non-vanishing dimension is a hint that the gravity should enter in a locally supersymmetric theory. The gauge field of the local supersymmetry is a (real) Majorana field since $\delta\psi_m \sim \partial_m\epsilon$ and ϵ is a Majorana spin-1/2 parameter. Since the Noether supercurrent carries dimension 7/2 we obtain the Noether coupling term $\kappa(\bar{\psi}_m J_N^m)$. Having introduced new fermionic degrees of freedom we must also introduce more bosonic degrees of freedom to balance them so as to main tain fermi-bose supersymmetry. This also follows from the fact that the supersymmetry current transforms into the strees tensor T^{lm} of the matter system. The coupling $\bar{\Psi}_m J^m_N$ thus requires at the same time a term $\kappa h_{lm} T^{lm}$ where $h_{lm} = h_{ml}$ with dimension 1 is a new bosonic (spin 2) compensating gauge field (of linearized gravity). The gauge fields of local supersymmetry are thus suggested to belong to the (2,3/2) supergravity supermultiplet of N=1 supersymmetry rather than the (3/2,1) supermultiplet. We remark that the gravity (spin 2) is necessarily coupled to stress tensor of all matter while neither the real spin 3/2 field nor any other real matter field can couple minimally to spin 1 photon. The gauge field ψ_m being a superpartner of spin-2 graviton should describe a spin 3/2 particle which is called 'gravitino'. The simplest locally supersymmetric theory contains just these two fields and will be described below after a brief sketch of the tetrad formulation of ordinary gravity which is needed since we have fermions in the theory.

2 TETRAD FORMULATION OF EINSTEIN-CARTAN THEORY OF GRAVITATION[4]

The space-time manifold is labelled by coordinates x^μ where $\mu = 0,1,2,3$ indicates the world index. We may introduce at each space-time point a sufficiently differentiable field of four vectors (vierbein or tetrad frame) $\underline{e}_m = e^\mu_m \partial_\mu$ where $m = 0,1,2,3$ and $[e^m_\mu] = 0$. We assume also the existence of a constant Minskowski metric* η_{mn} and choose the tetrads to be orthonormal, $\underline{e}_m \cdot \underline{e}_n = \eta_{mn}$.

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* $\eta_{\ell m} = \text{diag}(-1,1,1,1)$.

We have the dual frame $\underline{e}^m = e^m_\mu dx^\mu$ and find $e^n_\mu e^\mu_m = \delta^\mu_\nu e^n_\mu e^\nu_m = \delta^n_m$ which implies $e^m_\mu e^\nu_m = \delta^\nu_\mu$. We may define anholonomic components of a tensor field reffered to a tetrad basis, for example, $A^m = e^m_\mu A^\mu$, $\eta_{nm} = e^\mu_n e^\nu_m g_{\mu\nu}$, $A^m B_m = e^m_\mu A^\mu e^\nu_m B_\nu = A^\mu B_\mu$ etc. The general coordinate transformations $x_\mu \to x'_\mu$ or diffeomorfisms keep the local tetrad frames fixed while the local Lorentz transformations describe the rotations of the tetrad frames, independently from each other at each point x. The tetrad fields $e^m_\mu(x)$ are supposed to describe gravitation. Under a combined infinitesimal transformation we find

$$\delta e^m_\mu = \zeta^\alpha(x)\partial_\alpha e^m_\mu + (\partial_\mu \zeta^\nu)e^m_\nu + \lambda^m{}_n e^n_\mu \quad ,$$

$$\delta e^\mu_m = \zeta^\alpha(x)\partial_\alpha e^\mu_m - (\partial_\nu \zeta^\mu)e^\nu_m - \lambda^n{}_m e^\mu_n \qquad (3)$$

where $x'_\mu = x_\mu - \zeta_\mu(x)$ and the Lorentz rotation parameters satisfy $\lambda^m{}_n = -\lambda_n{}^m$, $\lambda^m{}_n = (i/2)\lambda_{pq}(M^{pq})^m{}_n$ where M_{mn} are the Lorentz generators. In order to define covariant derivative of an object like e^m_μ with mixed types of indices we need two kinds of connections, connection $\Gamma_{\mu\nu}{}^\rho$ to differentiate the world indices and connection $\omega_\mu{}^\ell{}_m$ to differentiate the local frame or tangent space indices. We define to set up our notation

$$D_\mu e^m_\nu = \partial_\mu e^m_\nu - \Gamma_{\mu\nu}{}^\alpha e^m_\alpha + \omega_\mu{}^m{}_n e^n_\nu \qquad (4)$$

and it follows from $D_\mu(e^m_\rho e^\nu_m) = 0$ that

$$D_\mu e^\nu_m = \partial_\mu e^\nu_m + \Gamma_{\mu\alpha}{}^\nu e^\alpha_m - \omega_\mu{}^n{}_m e_n{}^\nu \qquad (5)$$

while $[\Gamma] = [\omega] = 1$. The covariant derivatives of other tensors are defined analogously. Since the local components A^m of a vector A^μ are world scalars, $\delta_{gc} A^m = \zeta^\alpha \partial_\alpha A^m$ and the covariant derivative $D_\mu A^m = \partial_\mu A^m + \omega_\mu{}^m{}_n A^n$ by definition transforms as a good tensor on all its indices it follows that the index μ in $\omega_\mu{}^m{}_n$ is tensorial and

$$\delta_{gc}\omega_\mu{}^m{}_n = \zeta^\alpha \partial_\alpha \omega_\mu{}^m{}_n + (\partial_\mu \zeta^\nu)\omega_\nu{}^m{}_n \tag{6}$$

Under the Lorentz rotations the world index is inert and we find

$$\lambda^m{}_n D_\mu A^n = \partial_\mu(\lambda^m{}_n A^n) + (\delta_L \omega_\mu{}^m{}_n)A^n + \omega_\mu{}^m{}_n \lambda^n{}_p A^p \tag{7}$$

which leads to

$$\delta_L \omega_\mu{}^m{}_n = -D_\mu \lambda^m{}_n = -\partial_\mu \lambda^m{}_n + \lambda^m{}_p \omega_\mu{}^p{}_n - \lambda^p{}_n \omega_\mu{}^m{}_p \tag{8}$$

The presence of the inhomogeneous term shows that $\omega_\mu{}^m{}_n$ is a connection under Lorentz transformations. A similar discussion based on $D_\mu A^\nu = \partial_\mu A^\nu + \Gamma_{\mu\alpha}{}^\nu A^\alpha$ requires $D'_\mu A'^\nu(x') = \frac{\partial x'^\nu}{\partial x^\lambda} \cdot \frac{\partial x^\sigma}{\partial x'^\mu} D_\sigma A^\lambda(x)$ and leads to

$$\begin{aligned}\delta_{gc}\Gamma_{\mu\nu}{}^\lambda &= \Gamma'_{\mu\nu}{}^\lambda - \Gamma_{\mu\nu}{}^\lambda \\ &= \partial_\mu\partial_\nu\zeta^\lambda + \zeta^\alpha\partial_\alpha\Gamma_{\mu\nu}{}^\lambda + (\partial_\mu\zeta^\alpha)\Gamma_{\alpha\nu}{}^\lambda + (\partial_\nu\zeta^\alpha)\Gamma_{\mu\nu}{}^\lambda\end{aligned} \tag{9}$$

while

$$\delta_L \Gamma_{\mu\nu}{}^\lambda = 0 \tag{10}$$

It is not possible to recast (9) in a form analogous to (8) showing that the general coordinate transformations are distinct from the Lorentz rotations. We also note that the torsion $C_{\mu\nu}{}^{\lambda} = \frac{1}{2}(\Gamma_{\mu\nu}{}^{\lambda} - \Gamma_{\nu\mu}{}^{\lambda})$ transforms like a tensor while $\delta_L(\omega_\mu{}^{mn} + \omega_\mu{}^{nm}) = 0$ as expected also from $D_\mu \eta_{\ell m} = -(\omega_{\mu\ell m} + \omega_{\mu m\ell})$ and $\delta_L \eta_{\ell m} = 0$. From the fact that $D_\mu e^m_\nu$ is a good tensor we may impose the metricity constraint $D_\mu e^m_\nu = 0$ which leads to $\Gamma_{\mu\nu}{}^{\rho} = e_m{}^{\rho}(\partial_\mu e^m_\nu + \omega_\mu{}^m{}_n e^n_\nu)$ and $D_\mu e_m^\nu = 0$ while reducing the number of independent fields. The metric tensor for the world indices is defined as the composite object $g_{\mu\nu} = \eta_{mn} e^m_\mu e^n_\nu$ and its inverse by $g^{\mu\alpha} g_{\alpha\nu} = \delta^\mu_\nu$. We then find $D_\lambda g_{\mu\nu} = -2\omega_{\lambda(mn)} e^m_\mu e^n_\nu$. The Einstein-Cartan geometry is defined by imposing the metricity postulate for $g_{\mu\nu}$, viz, $D_\lambda g_{\mu\nu} = 0$ which requires the antisymmetry in the local indices of the spinor connection, $\omega_{\lambda mn} = -\omega_{\lambda nm}$, so that $D_\lambda \eta_{mn} = 0$. The space-time connection for this geometry may be shown to take following form

$$\Gamma_{\mu\nu}{}^{\lambda} = \overset{0}{\Gamma}_{\mu\nu}{}^{\lambda} - K_{\mu\nu}{}^{\lambda} \tag{11}$$

where

$$\overset{0}{\Gamma}_{\mu\nu}{}^{\lambda} = \overset{0}{\Gamma}_{\nu\mu}{}^{\lambda} = \frac{1}{2} g^{\lambda\alpha}[\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu}] \tag{12}$$

are the symmetric Christoffel connections with respect to which the metric tensor already satisfies the metricity condition while

$$-K_{\mu\nu}{}^{\lambda} = g^{\lambda\alpha}[g_{\alpha\beta} C_{\mu\nu}{}^{\beta} + g_{\mu\beta} C_{\alpha\nu}{}^{\beta} + g_{\nu\mu} C_{\alpha\mu}{}^{\beta}] \tag{13}$$

is the contorsion tensor with the symmetry $K_{\mu\nu}{}^{\lambda} = -K_\mu{}^\lambda{}_\nu$ and which vanishes when torsion is zero. For the Riemannian geometry with vanishing torsion the metricity condition on the tetrad gives the following spinor connection

$$\overset{\circ}{\omega}_{\mu}{}^{\ell m} = \frac{1}{2}\,[e^{\ell\nu}(\partial_{\mu}e^{m}_{\nu} - \partial_{\nu}e^{m}_{\mu}) + \frac{1}{2}\,e^{\ell\lambda}e^{m\sigma}(\partial_{\sigma}e_{\lambda n} - \partial_{\lambda}e_{\sigma n})\,e^{n}_{\mu}] - (\ell \leftrightarrow m) \tag{14}$$

while for the Einstein-Cartan space-time we find

$$\omega_{\mu}{}^{mn} = \overset{\circ}{\omega}_{\mu}{}^{mn} + K_{\mu}{}^{mn} \tag{15}$$

where $K_{\mu}{}^{mn} = -K_{\mu}{}^{nm} = K_{\mu\alpha}{}^{\beta}e^{\alpha m}e_{\beta}{}^{n}$. We also note that

$$\mathcal{D}_{\mu}e^{m}_{\nu} - \mathcal{D}_{\nu}{}^{m}_{\mu} = 2C_{\mu\nu}{}^{\alpha}e^{m}_{\alpha} \tag{16}$$

where $\mathcal{D}_{\mu}e^{m}_{\nu} = \partial_{\mu}e^{m}_{\nu} + \omega_{\mu}{}^{m}{}_{n}\,e^{n}_{\nu} = [\delta^{m}{}_{n}\partial_{\mu} + \frac{i}{2}\,\omega_{\mu pq}(M^{pq})^{m}{}_{n}]\,e^{n}_{\nu}$ is the non-minimal covariant derivative.

The space-time curvature tensor and the Lorentz curvature tensor may be conveniently introduced by considering $[D_{\rho},D_{\lambda}]$ acting on an arbitrary tensor A^{m}_{μ}. When acting on the vierbein fields we find the relation[5]

$$[D_{\rho},D_{\lambda}]e^{m}_{\mu} + 2C_{\rho\lambda}{}^{\alpha}D_{\alpha}e^{m}_{\mu} = R^{\alpha}{}_{\mu\rho\lambda}(\Gamma)\;e^{m}_{\alpha} - R^{m}{}_{n\rho\lambda}(\omega)e^{n}_{\mu} \tag{17}$$

where

$$-R^{\mu}{}_{\nu\rho\lambda}(\Gamma) = \partial_{\rho}\Gamma_{\lambda\nu}{}^{\mu} + \Gamma_{\rho\alpha}{}^{\mu}\Gamma_{\lambda\nu}{}^{\alpha} - (\rho \leftrightarrow \lambda)\;,$$

$$-R^{m}{}_{n\rho\lambda}(\omega) = \partial_{\rho}\omega_{\lambda}{}^{m}{}_{n} + \omega_{\rho}{}^{m}{}_{p}\omega_{\lambda}{}^{p}{}_{n} - (\rho \leftrightarrow \lambda)$$

$$= \{\partial_{\rho}\omega_{\lambda} - \partial_{\lambda}\omega + [\omega_{\rho},\omega_{\lambda}]\}^{m}{}_{n} \equiv \{F_{\rho\lambda}(\omega)\}^{m}{}_{n} \tag{18}$$

with $\omega_\lambda = (\omega_\lambda{}^m{}_n)$. On imposing the metricity condition for the tetrads we obtain

$$R^\mu{}_{\nu\rho\lambda}(\Gamma) = R^m{}_{n\rho\lambda}(\omega) e^n_\nu e^\mu_m \tag{19}$$

The curvature scalar $R(g,\Gamma)$ may then be expressed as $R(e,\omega)$ where

$$R(e,\omega) = \frac{1}{2} H^{\lambda\rho}_{mn}(e) R^{mn}{}_{\rho\lambda}(\omega) \quad ,$$

$$H^{\lambda\rho}_{mn}(e) = e^\lambda_m e^\rho_n - e^\rho_m e^\lambda_n = \frac{1}{2} e^p_\mu e^q_\nu \varepsilon_{pqmn}(\frac{\varepsilon^{\mu\nu\lambda\rho}}{e}) \tag{20}$$

where $e = \det(e^m_\mu) = 1/\det(e^\mu_m) = (-\det(g_{\mu\nu}))^{1/2}$ is scalar density of weight +1.

The Lagrangian for the gravitational action $\simeq (-g)^{1/2} R(g,\Gamma)$ in the first order form may then be written in terms of e^m_μ and $\omega_{\lambda mn}$ as

$$L = -\frac{1}{2\kappa^2} e R(e,\omega) = \frac{1}{8\kappa^2} e^p_\mu e^q_\nu \varepsilon_{pqmn} \varepsilon^{\mu\nu\lambda\rho} R^{mn}{}_{\lambda\rho}(\omega) \tag{21}$$

where $M = \frac{1}{\kappa} = M_{Pl}/(8\pi)^{1/2} \simeq 2.4 \times 10^{18}$GeV. The equations of motion are obtained through Palatini's procedure by varying e^m_μ and $\omega_{\lambda mn}$ independently. The variation* with respect to tetrad leads to

$$e R^n_\sigma - \frac{1}{2} e^n_\sigma R = \kappa^2 . \tau^n_\sigma \tag{22}$$

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*We note

$$e\ \varepsilon_{\mu\nu\rho\lambda} = e^m_\mu e^n_\nu e^p_\rho e^q_\lambda \varepsilon_{mnpq} \quad , \quad \frac{1}{e} \varepsilon^{\mu\nu\rho\lambda} = e^\mu_m e^\nu_n e^\rho_p e^\lambda_q \varepsilon^{mnpq} \quad ,$$

$$e\ \varepsilon_{\mu\nu\rho\lambda} e^\rho_p = e^m_\mu e^n_\nu e^q_\lambda \varepsilon_{mnpq} \quad , \quad \frac{1}{e} \varepsilon^{\mu\nu\rho\lambda} e^n_\nu = e^\mu_m e^\rho_p e^\lambda_q \varepsilon^{mnpq} .$$

where $R^{n}{}_{\sigma}(e,\omega) = R^{nm}{}_{\rho\sigma}(\omega)e^{\rho}_{m}$ and the energy-momentum tensor τ^{m}_{μ} is defined through the variation in the action for the matter I_M

$$\delta I_M = \int d^4x \ \tau^{m}_{\mu} \ \delta e^{\mu}_{m} \tag{23}$$

in analogy to the definition in the case of matter with integral spins of the symmetric energy momentum tensor $\delta I_M/\delta g_{\mu\nu} = -(1/2)\tau^{\mu\nu}$. With regard to the variation of $\omega_{\lambda mn}$ we derive easily the Palatini identity

$$-\delta R^{mn}{}_{\rho\lambda} = \mathcal{D}_{\rho}\delta\omega_{\lambda}{}^{mn} - \mathcal{D}_{\lambda}\delta\omega_{\rho}{}^{mn} \tag{24}$$

where $\delta\omega_{\lambda mn}$ is a good tensor and after dropping the surface terms we find

$$\frac{1}{2}\,\varepsilon_{pqm\ell}\,\varepsilon^{\mu\nu\lambda\rho}\,e^{q}_{\nu}[\mathcal{D}_{\rho}(\omega)\,e^{p}_{\mu} - \mathcal{D}_{\mu}(\omega)\,e^{p}_{\rho}] = \kappa^2\zeta^{\lambda}_{m\ell} \tag{25}$$

where the spin density $\zeta^{\lambda}_{m\ell}$ is defined by $\delta I_M = (1/2)\int d^4x \ \zeta^{\lambda}_{m\ell}\,\delta\omega_{\lambda}{}^{m\ell}$. We notice that only the non-minimal covariant derivative appears in (25) and if we use the metricity postulate for the tetrad this field equation reduces to an algebraic equation in view of (16) relating torsion with the spin density of the matter fields other than the gravitation.

We remark finally that the gravitino field ψ_μ transforms as a spinor under the Lorentz rotations while as a vector under the general coordinate transformations because spinors are world scalars

$$\delta\psi_\mu = \zeta^{\alpha}\partial_{\alpha}\psi_\mu + (\partial_\mu\zeta^{\nu})\psi_\nu + (i/2)\lambda_{mn}M^{mn}\psi_\mu \tag{26}$$

where $iM_{mn} = (1/4)[\gamma_m, \gamma_n]$ are the generators of spin-1/2 field. The action for the gravitino will be discussed in the next Section.

3 LAGRANGIAN FOR N=1 SUPERGRAVITY[6]

The simplest theory of pure supergravity may be formulated in terms of the vierbein field and a Rarita-Schwinger spin-3/2 field ψ_μ. The coupling of the latter to the gravitation however, must be most or non-minimal in order to preserve the gauge invariance constraint $\delta\psi_\mu = \partial_\mu \alpha$ of the free Rarita-Schwinger action analogous to the most minimal coupling of the Maxwell field to gravity. The supergravity Lagrangian in the second order formulation is given by

$$L_{SG} = -\frac{1}{2\kappa^2} eR(e,\omega(e,\psi)) - \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu (\mathcal{D}_\rho \psi_\sigma - \mathcal{D}_\sigma \psi_\rho) \tag{27}$$

where

$$\omega_{\mu\ell m} = \mathring{\omega}_{\mu\ell m}(e) + K_{\mu\ell m}(\psi) = \omega_{\mu\ell m}(e,\psi) \quad ,$$

$$K_{\mu\ell m} = \frac{i}{4} \kappa^2 (\bar{\psi}_\mu \gamma_\ell \psi_m - \bar{\psi}_\mu \gamma_m \psi_\ell + \bar{\psi}_\ell \gamma_\mu \psi_m) \quad , \tag{28}$$

$\bar{\psi}_\mu = \psi^+_\mu \gamma^{(0)}$ where $\gamma^{(0)} = (\gamma^\ell)_{\ell=0}$ is constant matrix and† $\mathcal{D}_\rho = (\partial_\rho + \frac{1}{2} \omega_{\rho\ell m} \gamma^{\ell m})$ indicates the non-minimal covariant derivative acting on spin-1/2 fields. The curl $\mathcal{D}_{[\rho}\psi_{\sigma]}$ is covariant like the curl $\partial_{[\mu}A_{\nu]}$. Each term of the action may be shown to be invariant under diffeomorfisms as well as under local Lorentz rotations. The minimal action

† $\gamma^{\ell m} = \frac{1}{4} [\gamma^\ell, \gamma^m] = iM^{\ell m}$.

considered here is the complete action for simple supergravity in four dimensional space-time. In five and eleven dimensions one needs extra four-fermion couplings and also anti-symmetric tensor fields. The supergravity action above may be shown to be invariant under the following local supersymmetry transformations

$$\delta e^{\ell}_{\mu} = \frac{\kappa}{2}\,\bar{\epsilon}(x)\gamma^{\ell}\psi_{\mu} \quad , \quad \delta\psi_{\mu} = \frac{1}{\kappa}\,D_{\mu}\epsilon(x) \tag{29}$$

where $D_{\mu}\epsilon = (\partial_{\mu} + \frac{1}{2}\omega_{\mu\ell m}(e,\psi)\gamma^{\ell m})\epsilon$. These transformation laws as well as the Lagrangian can be derived by an iterative procedure in the gravitational constant by starting with the free Lagrangian ($\kappa = 0$). The linearized vierbein can be written as $e_{\mu\ell} = \eta_{\mu\ell} + \kappa\, h_{\mu\ell}$ where $h_{\mu\ell} = h_{\ell\mu}$ describes the free graviton spin-2 field. For $\kappa=0$ the Lagrangian is invariant under two separate abelian gauge transformations

$$\delta h_{\mu\nu} = \partial_{\mu}\zeta_{\nu}(x) + \partial_{\nu}\zeta_{\mu}(x) \quad ; \quad \delta\psi_{\mu} = \partial_{\mu}\alpha(x) \tag{30}$$

and the following rigid supersymmetry transformations

$$\delta h_{\mu\nu} = \bar{\epsilon}\gamma_{\mu}\psi_{\nu} + \bar{\epsilon}\gamma_{\nu}\psi_{\mu} \quad , \quad \delta\psi_{\mu} = \frac{1}{2}\,\mathcal{B}_{\mu\ell m}\gamma^{\ell m}\epsilon \tag{31}$$

We may use Noethers step by step procedure mentioned before to arrive at the non-linear Lagrangian along with the local supersymmetry transformation laws. We note that at the coupled level the two abelian transformations independent at the linearized level combine into an irreducible non-abelian local s.s. transformation law. This is analogous to the case of the local Yang--Mills transformation $\delta v^{a}_{\ell} = D_{\ell}\Lambda^{a} = \partial_{\ell}\Lambda^{a} + f^{a}_{\ bc}v^{b}_{\ell}\Lambda^{c}$ which at the linearized level also splits into an abelian gauge transformation and a global Yang-Mills rotation.

The local algebra, e.g., the commutator of two local symmetries may be calculated straightforwardly. We find, for example, that as in the case of rigid supersymmetry in the local case as well the commutator $[\delta_s, \delta_s]$ when acting on the fermionic field ψ_μ generates a term proportional to the gravitino field equations indicating the need for introducing auxiliary fields in the theory which are known for simple supergravity in the second order formulation. Moreover, along with producing the expected general coordinate transformation there are also found terms on the r.h.s. which represent a local Lorentz and a local supersymmetry transformation. We will not dwell on these details and remark only that including auxiliary fields, a scalar, a pseudoscalar and an axial vector A^μ, the local algebra in the second order formulation does 'close' contrary to the case of the first order formulation. The supergravity theory may also be shown to be a gauge theory of the super-Poincaré group and may also be derived as the geometry of superspace.[7] Corresponding to the graded conformal group a conformal supergravity theory can also be constructed.

4 SUPERGRAVITY COUPLING TO MATTER

4.1 Non-linear realization of supersymmetry. Coupling of Volkov--Akulov field

When the global supersymmetry is realized in the spontaneously broken mode the resulting 'goldstino' field λ corresponding to the broken N=1 supersymmetry has a non-linear s.s. transformation law along with an inhomogeneous term given by[8]

$$\delta\lambda = d\epsilon + \frac{i}{d} (\bar{\lambda}\gamma^{\ell}\epsilon)\partial_{\ell}\lambda \qquad (32)$$

where ϵ and λ are Majorana spinors and we use four component notation. The unbroken Poincaré transformations are, however, realized linearly on λ. This is analogous to the case of original sigma model where the massless Goldstone pions transform non-linearly under the broken SU(2) generators while linearly under the unbroken SU(2) and the effective low energy Lagrangian is a non-linear model. The constant 'd' of dimension 2 indicates the square of the s.s. breaking scale and $\langle\delta\lambda\rangle_0 = d.\epsilon \neq 0$ indicates that the supersymmetry is broken. It is easily shown that the above non-linear transformation closes into the supersymmetry algebra .

$$[\delta_2,\delta_1]\lambda = -2i(\bar{\epsilon}_2\gamma^{\ell}\epsilon_1)\partial_{\ell}\lambda \qquad (33)$$

No other fields are needed to make the realization faithful. We note also that if $\rho(x)$ is another field with the homogeneous transformation law $\delta\rho = \frac{i}{d}(\bar{\lambda}\gamma^{\ell}\epsilon)\partial_{\ell}\rho(x)$ then the algebra closes on ρ as well. The simplest non-linear Lagrangian invariant under supersymmetry up to a divergence is given by

$$L_{\lambda} = -\frac{d^2}{2}\det(\delta^{\ell}_m - \frac{i}{d^2}\bar{\lambda}\gamma^{\ell}\partial_m\lambda) = -\frac{d^2}{2} + \frac{i}{2}\bar{\lambda}\gamma^{\ell}\partial_{\ell}\lambda + \ldots + 0(d^{-6}) \qquad (34)$$

where the dots represent the interaction terms $\frac{1}{2}(T^m{}_m T^n{}_n - T_m{}^n T^m_n)$ + $+ 0(T^3) + 0(T^4)$ where $T_m{}^n = \frac{i}{d^2}\bar{\lambda}\gamma_m\partial^n\lambda$ and the serie terminates since $\lambda(\bar{\lambda}\lambda)^2 = 0$. The Noether supercurrent is derived to be

$$J_m = id\gamma_m\lambda + \ldots \quad , \quad \partial_m J^m \stackrel{\circ}{=} 0 \qquad (35)$$

and $\langle 0|J^{\ell}_{A}|\lambda_B\rangle = id(\gamma^{\ell})_{AB}$. The positive vacuum energy density $d^2/2$ shows again that the supersymmetry is broken.

When the supersymmetry is promoted to be local one a non-diagonal term $\kappa\bar{\psi}_{\ell}J^{\ell}_{N} = \kappa d(\bar{\psi}.\gamma\lambda)$ will be added to the Lagrangian, say, if we follow the Noether coupling procedure. Deser and Zumino[b] studied the coupling of Volkov-Akulov Lagrangian to de Sitter supergravity[§] and showed that under the assumption of a vanishing cosmological term, the gravitino acquires a mass $\sim d^2$ while the goldstino field can be absorbed into a redefinition of the fields e^{m}_{μ} and ψ_{μ}. The super-Higgs effect for general interaction of the chiral supermultiplets and gauge supermultiplet will be described in the following sections. We remark that the Higgs mechanism for gauged supersymmetry algebra is distinct from the same mechanism for the gauged Lie algebras owing to certain positivity properties of superalgebras.[9] The supersymmetry restricts the scalar field content which determines the spontaneous breaking and the vacuum energy. The overall scalar potential has a unique form once the supersymmetry variations of the fermionic fields in the theory are given.

[§] The V-A Lagrangian gives rise to a negative cosmological term $-d^2e/2$. We may add to the Lagrangian a supersymmetric cosmological term $3m^2e + ime\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}$ with the positive cosmological constant while slightly modifying the transformation laws to have a local s.s. invariance. The vanishing of the cosmological constant in the total Lagrangian determines $m = d$. Choosing the gauge $\lambda = 0$ (or alternatively redefining the gravitino field through a local s.s. transformation with parameter λ which eliminates the propagation of the goldstino field) we obtain in the theory besides gravitation a massive gravitino apart from the interaction terms.

5 GENERAL COUPLING TO YANG-MILLS THEORY[10]

The N=1 pure supergravity Lagrangian can be coupled in a locally supersymmetric fashion to an arbitrary Yang-Mills system which is specified by the gauge group K and by the transformation properties under K of the set of chiral matter supermultiplets. The presence of the dimensional coupling-constant κ in pure supergravity Lagrangian leads to a non-renormalizable theory even if the matter-gauge system coupled to it is renormalizable one. We should rather demand instead that after the coupling to supergravity the resulting non-renormalizable terms are such that in the flat space limit, $M_{Pl} \to \infty$, the theory becomes renormalizable. In supergravity a spontaneously broken, locally supersymmetric theory, admits as a global limit, an explicitly broken, supersymmetric theory with soft s.s. breaking terms. The most complete form of the interacting theory was given by Cremmer et al. and we will adopt their notation. The component fields of left-handed chiral supermultiplets S_i transforming according to representation R of the gauge group K will be indicated by (z_i, χ_{Li}, h_i) where i labels the representation index, $T_i^{\alpha j}$ stand for the generators in the representation and α is the group index labelling the adjoint representation. The gauge fermions are called λ^α while $F^\alpha_{\mu\nu}$ indicate the field strengths of the gauge bosons.

The arbitrariness of the interacting supergravity-Yang-Mills Lagrangian consists essentially in a non-canonical modification of the chiral and Yang Mills kinetic terms. They involve respectively a real gauge invariant function $G(z,z^*)$, called Kähler potential, and an analytic (chiral) function of the complex scalar fields z_i written $f_{\alpha\beta}(z)$ which transforms as the symmetric product of the adjoint representation of K. The requirement of local supersymmetry in its turn proliferates them in the interaction terms as well as in the local s.s. transformation laws. The scalar field in supergravity theories appear as coordinates of a Kähler manifold whose metric enters in the scalar kinetic terms which has a form characteristic of supersymmetric non-linear sigma model. The final Lagrangian has the following form

$$L = L_{BK} + L_P + L_{FK} + L_{FM} + L_{(4)F} \qquad (36)$$

where the bosonic part is given by ($\kappa=1$)

$$e^{-1} L_{BK} = -\frac{1}{2} R - G^i{}_j D_\mu z_i D^\mu z^{*j} - \frac{1}{4} \mathrm{Re} f_{\alpha\beta} F^\alpha_{\mu\nu} F^{\mu\nu\beta} + \frac{i}{4} \mathrm{Im} f_{\alpha\beta} F^\alpha_{\mu\nu} \tilde{F}^{\mu\nu\beta} ,$$

$$-e^{-1} L_p = V(z,z^*) = e^G [G^i (G^{-1})_i{}^j G_j - 3] + \frac{1}{2} \mathrm{Re}(f^{-1}_{\alpha\beta} D^\alpha D^\beta) = V_c + V_g \qquad (37)$$

where V_c and V_g are the chiral and gauge parts of the scalar potential respectively. The covariant derivatives are covariant w.r.t. the gravity and the gauge group and we define $G^i = \partial G/\partial z_i$, $G_i = \partial G/\partial z^{*i}$, $G^i{}_j = \partial^2 G/\partial z_i \partial z^{*j}$, $D_\alpha = g_\alpha G^i (T_\alpha)_i{}^j z_j$ with g_α indicating the gauge coupling constant associated to the normalized generators. We note that in the positivity domain of the spin-1 kinetic term the gauge field contribution V_g is a semipositive definite function. We will describe latter the necessary and sufficient conditions to obtain a semipositive definite V_c. The fermionic kinetic and mass terms are listed below while for the $L_{(4)F}$ we refer the reader to the original reference where the s.s. transformation laws are also given in complete form[11]

$$e^{-1}L_{FK} = -\frac{1}{4}\ \mathrm{Re} f_{\alpha\beta}\bar{\lambda}^{\alpha}\not{D}\lambda^{\beta} - \frac{1}{4}\ e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_5\gamma_{\nu}D_{\rho}\psi_{\sigma}$$

$$-G^{i}{}_{j}\bar{\chi}_{Li}\not{D}\chi_{R}{}^{j} - \frac{i}{8}\ \mathrm{Im}\, f_{\alpha\beta}e^{-1}D_{\mu}(e\bar{\lambda}^{\alpha}\gamma_5\gamma^{\mu}\lambda^{\beta}) + \text{h.c.}$$

$$e^{-1}L_{FM} = e^{G/2}\bar{\psi}_{\mu R}\sigma^{\mu\nu}\psi_{\nu R} + \bar{\psi}_{R}\cdot\gamma[e^{G/2}G^{i}\chi_{Li} - \frac{i}{2}\ \tilde{g}\, G^{i}T_{i}^{\alpha j}z_{j}\lambda_{L}^{\alpha}]$$

$$-e^{G/2}\bar{\chi}_{Li}[G^{ij} + G^{i}G^{j} - G^{\ell}G_{\ell}{}^{-1k}G_{k}{}^{ij}]\chi_{Lj} + 2i\tilde{g}\,G_{i}^{k}z^{*j}\,T_{j}^{\alpha i}\bar{\lambda}_{L}^{\alpha}\,\chi_{Lk}$$

$$+ \frac{1}{2}\,f_{\alpha\beta}^{k}\{\frac{1}{2}\,e^{G/2}G_{\ell}G_{k}^{-1\ell}\bar{\lambda}_{L}^{\alpha} - i\tilde{g}\ \mathrm{Re}\, f_{\alpha\gamma}^{-1}G^{i}\,T_{i}^{\gamma j}\,z_{j}\bar{\chi}_{Lk}\}\,\lambda_{L}^{\beta} + \text{h.c.} \tag{38}$$

where D_{μ} is covariant both with respect to the gauge group and gravitation. We list also the scalar field contributions to the fermionic local s.s. variations of the fermion fields

$$\delta\lambda_{R}^{\alpha} = +\frac{i}{2}\ \epsilon_{R}\,\mathrm{Re}f_{\alpha\beta}^{-1}D_{\beta} + \ldots\ ,$$

$$\delta\chi_{Li} = -\frac{1}{2}\ \epsilon_{L}\ e^{G/2}(G^{-1})_{i}{}^{j}\,G_{j} + \ldots$$

$$\delta\psi_{\mu} = \frac{1}{2}\ \gamma_{\mu}\epsilon_{R}\ e^{G/2} + \ \ldots \tag{39}$$

We remark that in arriving at the above Lagrangian it is necessary that G has the following form[§]

$$G(z,z^{*}) = J(z,z^{*}) + \ln|g(z)|^{2} \tag{40}$$

where the superpotential g(z) is a non-vanishing analytic function of z_i. We note that J and g are defined upto a Kähler trans

§When g=0 proper substitution rules must be used to obtain the correct Lagrangian.

formation

$$J \longrightarrow J + f(z) + f^*(z^*) \;, \qquad g(z) \longrightarrow g(z)e^{-f(z)} \tag{41}$$

The Lagrangian above is also implied in a superfield formulation where the extension of the global supersymmetry action to local supersymmetry should look like

$$\int d^8z\ E[\Phi(s,\bar{s}\,e^{2v}) + \mathrm{Re}(\frac{1}{R}\,g(s)) + \mathrm{Re}(\frac{1}{R}\,f_{\alpha\beta}(s)\,W_a^{\alpha}\,\varepsilon^{ab}\,W_b^{\beta}) \tag{42}$$

where Φ, g, f are three input functions, R is the chiral scalar curvature superfield and E is the superspace determinant. In the complete Lagrangian Φ and g loose their independent meaning and enter only through G above where J is related to Φ. The goldstino field is uniquely identified in the local supersymmetry under consideration by the spin-1/2 fermion which couples to the gravitino gauge field in L_{FM}

$$\eta_L = -[e^{G/2}\,G^i\chi_{Li} - \frac{i}{2}D^{\alpha}\lambda_L^{\alpha}] \tag{43}$$

6 THE SUPER-HIGGS EFFECT IN N=1 SUPERGRAVITY

In the standard Higgs mechanism the local gauge invariance allows a spin-0 Goldstone boson, corresponding to a spontaneously broken rigid symmetry, to be rotated away such that the initially massless gauge boson with the states of helicity ±1 acquires a third helicity-0 state becoming consequently massive - the spontaneous breaking of a local gauge symmetry. The corresponding super-Higgs mechanism may be shown to occur and the problem of the apparent non--existence of the massless goldstino particle of spontaneously broken rigid supersymmetry is thus overcome in the context of local supersymmetry.

We observed earlier that V_g is semi-positive definite in the positivity domain of the kinetic term of the Yang-Mills field. The chiral part V_c of the scalar potential is the difference between two positive definite terms due to the positivity properties of the Kähler metric G^i_j which is present in the kinetic terms of the spin-0 and spin-1/2 particles of the chiral supermultiplets and as such may assume positive, negative or vanishing value. Thus we may obtain the super-Higgs effect with vanishing vacuum energy (Minkowski space) since it is no more an order parameter for broken local supersymmetry contrary to the case of rigid supersymmetry. Moreover broken supergravity is possible even in the presence of a single chiral supermultiplet in contrast to the global supersymmetry case.

Through the minimal coupling $\bar{\psi}_R \cdot \gamma \eta_L +$ h.c. the local supersymmetry allows the goldstino to be rotated away by a special choice of the supersymmetric gauge while the previously massless gravitino with the states of helicity ±3/2 acquires the states of helicity ±1/2 and becomes massive. A necessary and sufficient condition for spontaneously broken supersymmetry requires that one of the quantities

$$e^{G/2} G^i \quad , \quad D^\alpha = g_\alpha G^i (T^\alpha)_i{}^j z_j \tag{44}$$

is different from zero at the minimum of the scalar potential, e.g., $(\partial V/\partial z_i)_{z=z_o} = 0$. If we may arrange also a vanishing vacuum energy at the minimum, $V_o = V(z_o, z_o^*) = 0$, i.e. a vanishing cosmological constant, then Minskowski space is a solution of the vacuum field equations and on this background the gravitino mass has its usual meaning and it is given by the value of the Kähler potential $m_{3/2} = M e^{G/2}$ analogously to the manner the value $\langle H \rangle_o$ fixes the gauge boson mass.

In the absence of supersymmetry breaking, $G^i = 0$, $D^\alpha = 0$ the vacuum energy, $V_o = -3 M^4 e^G < 0$, is negative which corresponds to anti-de Sitter ($G \neq 0$) or Poincaré ($G = -\infty$) supergravity with multiplets degenerate in mass. The super-Higgs effect due to the broken supersymmetry on the other hand may occur in Minskowski, de Sitter or anti-de Sitter space and $V_o > 0$ always describes a broken supergravity.

We consider first the case of 'minimal' coupling of the supergravity to Yang-Mills system defined by

$$G^i{}_j = \delta^i_j \quad , \qquad f_{\alpha\beta} = \delta_{\alpha\beta} \tag{45}$$

so that all the kinetic terms are canonical and the Lagrangian depends only the superpotential $g(z)$. We have

$$G(z,z^*) = z_i z^{*i}/M^2 + \ln|g(z)|^2/M^6 \quad ,$$

$$G^i = M\, \partial G/\partial z_i = z^{*i}/M + M\, g^i/g \quad ,$$

$$m_{3/2} = M e^{G/2} = e^{zz^*/2M^2}\,(|g(z)|/M^2) \quad ,$$

$$V_c = e^{zz^*/M^2}\,[\,|D_{z_i} g|^2 - 3(|g(z)|^2/M^2)] \tag{46}$$

where $g^i = \partial g/\partial z_i$ and $D_{z_i} = \partial/\partial z_i + z^{*i}/M^2$ is the Kähler covariant derivative. The supersymmetry is broken if at least one of the Kähler covariant derivative of the superpotential is non-vanishing. When $M \to \infty$ with z_i fixed the gravity becomes unimportant leaving behind the familiar condition of the broken global supersymmetry.

In the simplest case of a constant superpotential $g = m^3$, $(D^\alpha = 0)$, we find

$$V = m^6 \, e^{z_i z^{*i}/M^2} \, . \, (zz^*/M^4 - 3/M^2) \qquad (47)$$

which has its stationary points at $z_i = 0$ and $z = 2M$. The supersymmetry is unbroken for $z_{io} = 0$ with $V_o = -3\, M^2 (m_{3/2})^2$. This point, however, corresponds to a local maximum of the potential. The other values on the contrary correspond to minima with some non-vanishing $D_z g = z^*/M^2$. The s.s. is broken with a negative vacuum energy $V_o = -e^2 m^6/M^2$ corresponding to anti-de Sitter vacuum with the 'apparent' gravitino mass em^3/M^2.

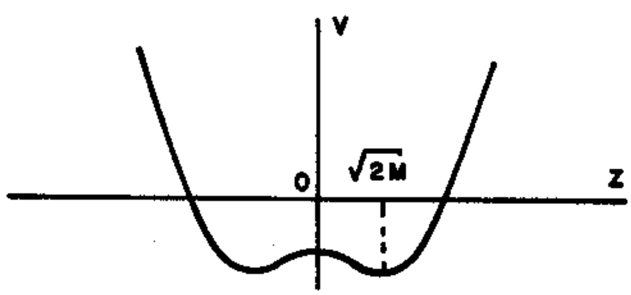

FIG. 1. Scalar potential with constant g for 'minimal' coupling.

The example shows that the supersymmetry may now be broken even in the presence of one matter supermultiplet and the supergravity breaking minima may be the lowest ones even if supersymmetric stationary points exists.

The Polonyi superpotential $g=m^2(z+\beta)$ offers itself as an example where we may fine tune the parameter β such that a broken supersymmetry minimum is obtained with vanishing vacuum energy, $V_{co}=0$. We find $D_z g = m^2[1+z^*(z+\beta)/M^2]$ and $D_z g = 0$ does not have any solution for z in the case $|\beta| < 2M$ and consequently the supersymmetry is certainly broken for such values of β. The chiral potential is given by

$$V_c = (\tfrac{m}{M})^4 \; e^{|z|^2/M^2} [|M^2+z^*(z+\beta)|^2 - 3M^2|z+\beta|^2] \tag{48}$$

It is straightforward to show that we can obtain absolute minimum with $V_c = 0$ at $z_o = \pm(\sqrt{3}-1)M$ if $\beta = \pm(2-\sqrt{3})M$ and where $D_z g = \sqrt{3}\, m^2$ with the gravitino mass given by

$$m_{3/2} = (m^2/M) \; e^{(\sqrt{3}-1)^2/2} \tag{49}$$

Performing the shift, say, $z \to z+(\sqrt{3}-1)M$ in V_c we may calculate the mass square of the real scalars which are found to be $2\sqrt{3}(m_{3/2})^2$ and $2(2-\sqrt{3})(m_{3/2})^2$. The scale of the supersymmetry breaking defined by the first term of the potential is given be

$$M_s^2 = \langle e^{zz^*/2M^2} |D_z g|\rangle_o = \sqrt{3}\, m^2 \; e^{(\sqrt{3}-1)^2/2} \tag{50}$$

and we find $M_s^2 = \sqrt{3}\, M\, m_{3/2}$. We note that the scalar field acquires a v.e.v. of the order of the Planck mass while the scalar masses are of the order of the gravitino mass and

$$m_{3/2} = \frac{M_s^2}{M_{Pl}} \sqrt{\frac{8\pi}{3}} \tag{51}$$

For the minimal coupling the term L_{FM} which determines the fermion mass matrix reduces to[12]

$$e^{-1}{}_{FM} = e^{G/2}[\bar{\psi}_{\mu R}\sigma^{\mu\nu}\psi_{\nu R} - \bar{\psi}_R\cdot\gamma\tilde{\eta}_L - \frac{2}{3}\bar{\tilde{\eta}}_L\tilde{\eta}_L]$$

$$+ \bar{\chi}_{Li}M^{ij}\chi_{Lj} + 2\bar{\chi}_{Li}M^{i\alpha}\lambda_L^{\alpha} + \bar{\lambda}_L^{\alpha}M^{\alpha\beta}\lambda_L^{\beta} + \text{h.c.} \qquad (52)$$

where $\tilde{\eta}_L = e^{-G/2}\eta_L$ and the spin-1/2 fermion matrix has the form

$$M^{ij} = -e^{G/2}[G^{ij} + \frac{1}{3}G^iG^j] \quad ,$$

$$M^{i\alpha} = -\frac{1}{3}G^iD^{\alpha} + i\,D^{\alpha i} \quad ,$$

$$M^{\alpha\beta} = -\frac{1}{6}\; e^{-G/2}\; D^{\alpha}D^{\beta} \qquad (53)$$

The quadratic mass relation may then be derived to be[§]

$$\text{Super trace } M^2 = \sum_{J=0}^{3/2}(-1)^{2J}(2J+1)m_J^2 = (N-1)(2m_{3/2}^2 - \kappa^2 D^{\alpha}D^{\alpha}) - 2\tilde{g}_{\alpha}D^{\alpha}\,\text{Tr}\,T^{\alpha} \qquad (54)$$

where the last term is only possible for Abelian U(1) factors of K with $\text{Tr}\,T^{\alpha} \neq 0$. If we set $m_{3/2}=0$ and $\kappa=0$ we get back the mass relations of spontaneously broken globally supersymmetric Yang-Mills theories. The above relation is a consequence of the simultaneous occurence of the Higgs and super-Higgs effects.

It has been recently realized that certain N=1 supergravity model theories may have semi-positive definite potential permiting

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§ See Ferrara, ref. 11 and the references cited therein.

them at the tree level a vanishing cosmological constants even in the presence of broken supersymmetry. A necessary and sufficient condition for having a semi-positive definite potential in the general case is given by $\det(-\phi) \leq 0$ where ϕ is the hermitian matrix $(\partial^2/\partial z_i z^{*j}) \exp(-G/3)$. In particular the necessary and sufficient condition for flat potential ($V_c \equiv 0$) is $\det(\phi)=0$.

The chiral potential may be re-written as follows, N denoting the number of chiral supermultiplets,

$$V_c = \frac{9}{N^2} e^{\frac{N+3}{3}} (G^{-1})_i{}^j \partial^i \partial_j e^{-\frac{N}{3} G} \tag{55}$$

The flatness of the potential requires a particular Kähelr potential such that

$$(G^{-1})_i{}^j \partial^i \partial_j e^{-\frac{N}{3} G} = 0 \tag{56}$$

A particular solution is

$$G = -\frac{3}{N} \sum_1^N \ell n[f_i(z) + f_i^*(z^*)] \tag{57}$$

which is equivalent to $G = -\frac{3}{N} \sum_1^N \ell n(z_i + z^{*i})$ upto field redefinitions $z \to f(z)$. The curvature tensor for the Kähler manifold $R^i{}_j$ is defined by[13]

$$R^i{}_j = \partial^i \partial_j \, \ell n \det(G^i{}_j) \tag{58}$$

For the above solution we obtain

$$R^i{}_j = \frac{2N}{3} G^i{}_j \tag{59}$$

indicating an Einstein manifold. However, this property alone is not enough to ensure the flatness of the potential. We remark that for flat potentials the vanishing of the cosmological constant occurs naturally at the classical level wheather the supersymmetry is broken or not. The gravitino mass in such models is undetermined and in such so called 'no-scale' models one hopes to get all the low energy scale parameters from the Planck mass through the radiative corrections and the mechanism of dimensional transmulation (and renormalization group equation). One of the main features of such supergravity models is the non-minimality of the kinetic terms of the scalars which form a non-compact symmetric Kähler manifold, viz, SU(1,1)/U(1) in the less symmetric case and SU(n,1)/ /SU(n)x U(1) in the maximally symmetric case where n here is the number of gauge non-singlet complex scalars fields. Such global non-compact groups play an essential role in $N \geq 4$ extended supergravity theories. The non-compact group invariance seeminhly guarenteeing a flat potential may be a relic of an underlying theory. In fact the effective potential derived from $E_8 x E_8$ superstring model[14] also has such a non-compact symmetry.

Consider the case of one scalar field with (κ=1)

$$G(z,z^*) = -3 \ell n (z+z^*) + \ell n|c|^2 \tag{60}$$

where g=c is a constant superpotential. We find $e^{G/2} G_z = -3|c|/(z+z^*)^{5/2}$, $m_{3/2} = e^{G/2} = |c|/(z+z^*)^{3/2}$, $R_{zz^*} = (2/3)\, G_{zz^*}$ and $V_c \equiv 0$ for any value of z. The supersymmetry is broken when $c \neq 0$ but the gravitino mass is non-vanishing but undermined. The Kähler manifold is an Einstein space with constant curvature and its isometries form a non-compact SU(1,1) group. The Lagrangian for the gauge singlet scalar z reads as

$$3\sqrt{-g}\ \frac{(\partial_\mu z)(\partial^\nu z^*)}{(z+z^*)^2}\ g^{\mu\nu} \tag{61}$$

which describes a non-linear sigma model with an SU(1,1)/U(1) global symmetry. Furthermore the SU(1,1) Möbius transformations

$$z \to \left(\frac{\alpha z+i\beta}{i\gamma z+\delta}\right) \qquad \alpha,\beta,\gamma,\delta \text{ real with } \alpha\delta+\beta\gamma=1 \tag{62}$$

leave the whole Lagrangian, except the gravitino-goldstino mass term, invariant after simultaneous chiral rotations on the fermionic fields. For c=0 the supersymmetry is not broken and $m_{3/2}=0$ while all SU(1,1) breaking terms drop out from the Lagrangian. For $c\neq 0$ the supersymmetry is spontaneously broken and simultaneously the SU(1,1) symmetry is broken down to an $U(1)_{NC}$ defined by the imaginary translations $z\to z+i\beta$.

The existence of non-compact and anomaly free global symmetry seems necessary along with a 'no-scale'[15] model to obtain a vanishing cosmological constant. The SU(n,1) 'no-scale' model is based on the following G

$$G = -3\ln\left(z+z^* - \frac{\phi_i\phi^{*i}}{3}\right) + \ln|g(\phi_i)|^2 \tag{63}$$

where the superpotential g depends only on the 'observalbe' (sector) gauge non-singlet chiral superfields ϕ_i while the singlet z belongs to the 'hidden' sector and has the form

$$g(\phi_i) = c + d_{ijk}\phi^i\phi^j\phi^k \tag{64}$$

The gauge singlet plays the similar role as in the simple case considered above and the parameter c breaks supersymmetry spontaneously. The scalars potential is positive semi-definite and flat along the directions $(3/2)^{1/2}i(z-z^*)$ and $-(3/\sqrt{6})\ln(z+z^*-\phi_i\phi^{*i}/3)$.

Recently, superstring models have been proposed to solve the problems of quantum gravity, of unifying all the fundamental interactions, of flavour and the cosmological constant. The effective low energy theory obtained from the ten dimensional E_8xE_8 superstring theory after the compactification of the extra six dimensions on a Ricci flat manifold is found to be based on the following Kähler potential[16]

$$G = -3\ln(z+z^*-\phi_i\phi^{*i}) + \ln|W(S,\phi_i)|^2 - \ln(S+S^*) \quad (65)$$

and a simple non-trivial chiral kinetic function for the gauge superfield strength W^α, $f_{\alpha\beta}WW$, $f_{\alpha\beta}=\delta_{\alpha\beta}S$. Here $W(S,\phi_i)$ is an effective S-dependent superpotential of another 'hidden' sector gauge singlet chiral superfield S which is generated by the 'hidden E_8' gaugino condensation and the gauge non-singlet observable fields ϕ_i. The resulting theory has a 'no-scale' (SU(1,1)/U(1))x x (SU(n,1)/SU(n)xU(1)) structure. The local s.s. breaking scale at the tree level is a non-vanishing (undetermined) gravitino mass, which has to be fixed by, say, a dynamical determination of S. The limiting low energy theory[17] is obtained by taking the flat limit, $M_{Pl}\to\infty$ with $m_{3/2}$ fixed and dropping out decoupled and superheavy fields. The renormalized non-minimal kinetic function for the gauge field, $f_{\alpha\beta}=(S+b\ \ln U(\phi_i))$, gives rise to a gaugino mass $m_{1/2}$ and it is argued currently that it is responsible for the dominant source of global supersymmetry breaking in the observable sector to produce via radiative corrections non-zero scalar masses (for sleptons, for example) for the gauge non-singlet fields ϕ_i. The supergravity theories may be then regarded as effective theories for the light particle states once integration over the infinitely many massive modes of the superstring spectrum has been performed.

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