ISSN 0029 - 3865

CBPF-NF-007/84

A HIGH TEMPERATURE INTERPARTICLE POTENTIAL FOR AN ALTERNATIVE GAUGE MODEL

by

R.M. Doria[1]*, J.A. Helayel-Neto[2] and Zhao Wanyun[2]**

[1]Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[2]International School for Advanced Studies (S.I.S.S.A.)
Trieste, Italy

*International Centre for Theoretical Physics (ICTP)
Trieste, Italy

**Permanent address: Institute of Theoretical Physics
Academia Sinica, Beijing, China

A HIGH TEMPERATURE INTERPARTICLE POTENTIAL FOR AN ALTERNATIVE GAUGE MODEL.

R.M. Doria

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

and

International Centre for Theoretical Physics (ICTP)
Trieste, Italy

J.A. Helayel - Neto

International School for Advanced Studies (S.I.S.S.A.)
Trieste, Italy

*Zhao Wanyun**

International School for Advanced Studies (S.I.S.S.A.)
Trieste, Italy

*<u>Permanent Address</u>: Institute of Theoretical Physics
Academic Sinica, Beijing, China

ABSTRACT

A thermal Wilson loop for a model with two gauge fields associated with the same gauge group is discussed. Deconfinement appears at high temperature. We cannot however specify the colour of the deconfined matter.

Key-word: High temperature.

1 INTRODUCTION

The advent of the quark physics brought a new experimental parameter referred to as colour. The experimental results have so far revealed colour in three types and with a confinement structure. This is bringing a new situation to physics. The most natural way that appears to understand such a situation is to observe regions where deconfinement can take place. This may be possible by exploring hadronic matter in wider environments, in particular at high temperatures or at high baryon densities.

QCD is being considered the theory to approach colour confinement. It shows that at high temperature thermal excitations produce a plasma of quarks and gluons which screens the colour electric flux [1]. The infrared behaviour of QCD at high temperature is controlled by the spacelike components of the gluon propagator, which develops a colour magnetic mass [2]. However, it is not experimentally clear which meaning the electric and magnetic masses would have. There is also a problem that, since the magnetic mass is of order g^2T (g is the gauge coupling constant and T the temperature), higher orders perturbative calculations give increasing infrared divergences. According to Ref.[7], the interquark potential at high temperature can be perturbatively evaluated only up to the order g^{10}.

The main effort in this paper is to investigate the possibility of obtaining deconfinement if one starts at zero temperature with a gauge invariant mass term for the gauge bosons. The whole approach is based on a gauge invariant Lagrangian involving two gauge fields associated with the same group. These fields, A_μ and B_μ, have the following transformation laws:

$$A_\mu \longrightarrow UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{1.a}$$

and

$$B_\mu \longrightarrow UB_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{1.b}$$

which yields the field-strength tensor $G^a_{\mu\nu}$ given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu B^a_\mu + [B_\mu, A_\nu]^a \tag{2}$$

One can therefore propose the following gauge Lagrangian

$$\mathcal{L}_G = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \frac{1}{2}M^2(A^a_\mu - B^a_\mu)^2 \tag{3}$$

The way of associating the two families of gauge fields, A^a_μ and B^a_ν, to the matter fields will depend on the building blocks which are adopted by the model. If the quarks are considered as such, then we associate different gauge fields to different quark families. Another class of model would be based on a kind of preonic structure called colorful stones [3]. There, fermionic and bosonic colour matter fields are associated to A^a_μ and B^a_ν respectively.

Our main purpose is the calculation, through the thermal Wilson loop, of the interparticle potential (quark-antiquark or stone-antistone) at high temperature the Lagrangian proposed in Eq. (3) gives rise to. This is done in Section 2. A section of conclusions follows where our main results are discussed (Section 3).

2 THE INTERPARTICLE POTENTIAL

The thermal Wilson loop, $<Tr\Omega(\vec{x})>$, can be taken as an order parameter to study the transition from a confined to an unconfined phase as the temperature increases. For the case of two gauge fields associated to a common gauge group, the thermal Wilson loop is given by

$$<Tr\Omega(\vec{x})> = <TrTexp\{ig \int_0^\beta d\tau\lambda^a [A_o^a(\tau,\vec{x}) + B_o^a(\tau,\vec{x})]\}>, \quad (4)$$

where it is assumed that the system is placed in a heat bath at a physical temperature $\frac{1}{\beta} = T$. The expectation value is taken over the gauge fields $A_\mu^a(\tau,\vec{x})$ and $B_\mu^a(\tau,\vec{x})$, whose dynamics is here assumed to be described by a pure Yang-Mills Lagrangian without additional matter fields. The integral over the Euclidean time in Eq. (4) is actually an integral over a closed path due to the periodic conditions $A_\mu^a(0,\vec{x}) = A_\mu^a(\beta,\vec{x})$ and $B_\mu^a(0,\vec{x}) = B_\mu^a(\beta,\vec{x})$. T stands for the time ordering and λ^a are the matrices representing the generators of the non-Abelian gauge group. We define the symbol $Tr = \frac{trace}{d(R)}$ in such a way that Tr I = 1, I being the identity matrix and d(R) the dimensionality of the representation R.

The termal Wilson loop is related to the free energy of an isolated colorful matter (relative to the volume).

$$e^{-\beta F_c} = <Tr\Omega(\vec{x})>, \quad (5)$$

where F_c is the free energy of the isolated colorful matter.

The free energy, $F_{c\bar{c}}$, of a colorless matter pair is given by

$$e^{-\beta F_{c\bar{c}}} = \langle Tr\Omega^{+}(\vec{x}).Tr\Omega(\vec{y})\rangle, \qquad (6)$$

where we refer to $F_{c\bar{c}} = V(\vec{x} - \vec{y})$ as the interaction potential for colour and anticolour matter. Here, it should be mentioned that we are considering the static potential approximation. The presence of two gauge fields leads automatically to the existence of three kinds of interparticle potentials: they are $F_{c_A\bar{c}_A}$, $F_{c_B\bar{c}_B}$ and $F_{c_A\bar{c}_B}$. They describe interactions with different mediating processes. For instance, the former involves only gauge fields of the type A^a_μ, whereas the latter involves gauge bosons of both kinds, A^a_μ and B^a_ν.

Since the running coupling constant, $g(T)$, vanishes as the temperature goes to infinity, perturbative calculations are reliable at sufficiently high temperature. We calculate below the two-point function of the thermal Wilson loop of Eq. (6) by means of a perturbative expansion.

$$\langle Tr\Omega^{+}(\vec{x}).Tr\Omega(\vec{y})\rangle = \langle Tr\bar{T}\exp\{-ig\int_0^\beta d\tau\lambda^a[A^a_0(\tau,\vec{x}) + B^a_0(\tau,\vec{x})]\}.$$

$$.TrT\exp\{ig\int_0^\beta d\tau'\lambda^{a'}[A^{a'}_0(\tau'_1\vec{\ }) + B^{a'}_0(\tau'_1\vec{y})]\}\rangle, \qquad (7)$$

where $\bar{T}$ means anti-time ordering prescription.

Notice also that

$$\langle Tr\Omega^{+}(\vec{x}).Tr\Omega(\vec{y})\rangle = \langle Tr\Omega^{+}(\vec{x})Tr\Omega(\vec{y})\rangle_c + \langle Tr\Omega^{+}(\vec{x})\rangle\langle Tr\Omega(\vec{y})\rangle, \qquad (6)$$

where the subscript c stands for the connected part of the two-point function of the thermal Wilson loop. Considering up to order g^4,

$$<Tr\Omega_A^+(\vec{x}).Tr\Omega_A(\vec{y})>_c = \frac{g^4}{4}Tr(\lambda^a\lambda^b)Tr(\lambda^{a'}\lambda^{b'}).$$

$$.<\bar{T}\int_0^\beta d\tau_1\int_0^\beta d\tau_2 A_0^a(\tau,\vec{x})A_0^b(\tau_2,\vec{x})T\int_0^\beta d\tau_1'\int_0^\beta d\tau_2' A_0^a(\tau_1',\vec{y})A_0^b(\tau_2',\vec{y})>. \quad (9)$$

Similarly $<Tr\Omega_B^+(\vec{x})Tr\Omega_B(\vec{y})>_c.<Tr\Omega_A^+(\vec{x})Tr\Omega_B(\vec{y})>_c$ and $<Tr\Omega_B^+(\vec{x})Tr\Omega_A(\vec{y})>_c$. The Feynman diagrams giving the lowest order contribution to these two-point functions are those depicted in Fig. 1.

Using the finite temperature Feynman rules, one can calculate that the graphs of Fig. (1b) and Fig. (1b*) contribute with the following expression

$$G_{1(A\to B)}(\vec{x}-\vec{y}) = g^4Tr(\lambda^a\lambda^b)Tr(\lambda^{a'}\lambda^{b'})\int_0^\beta d\tau_1'\int_0^\beta d\tau_2'\int_\beta^0 d\tau_1\int_\beta^{\tau_1} d\tau_2 .$$

$$.[\Delta_{a'b}(\tau_1'-\tau_2,\vec{y}-\vec{x})\Delta_{b'a}(\tau_2'-\tau_1,\vec{y}-\vec{x}) + \Delta_{a'a}(\tau_1'-\tau_1,\vec{y}-\vec{x})\Delta_{b'b}(\tau_1'-\tau_2,\vec{y}-x)], \quad (10)$$

where

$$\Delta_{a'a}(\tau_1'-\tau_2,\vec{x}-\vec{y}) = \frac{1}{\beta(2\pi)^3}\sum_n\int d^3\vec{k}.\exp\left[i\frac{2\pi}{\beta}n(\tau_1'-\tau_2) + i\vec{k}.(\vec{y}-\vec{x})\right]\frac{M^2\delta_{a'a}}{(\vec{k}^2+\omega_n^2)[\vec{k}+\omega_n^2+M^2]} \quad (11)$$

with

$$\omega_n = \frac{2\pi n}{\beta} .$$

Taking the Fourier transform,

$$G_{1(A\to B)}(\vec{p}) = N^*\int_0^\beta d\tau_1' \int_0^{\tau_1'} d\tau_2' \int_\beta^0 d\tau_1 \int_\beta^{\tau_1} d\tau_2 \cdot$$

$$\cdot \sum_{n,m}\int d^3\vec{k}\ \frac{e^{i\omega_n(\tau_1'-t_2)}e^{i\omega_m(\tau_2'-\tau_1)} + e^{i\omega_n(\tau_1'-\tau_1)}e^{i\omega_n(\tau_2'-\tau_2)}}{(\vec{k}^2+\omega_n^2)\,[(\vec{p}-\vec{k})^2+\omega_m^2]\,(\vec{k}^2+\omega_n^2+M^2)\,[(\vec{p}-\vec{k})^2+\omega_m^2+M^2]} \qquad (12)$$

where $N^* = \dfrac{g^4(N^2-1)M^4}{(2\pi)^6\beta^2(16N^2)}$ for SU(N).

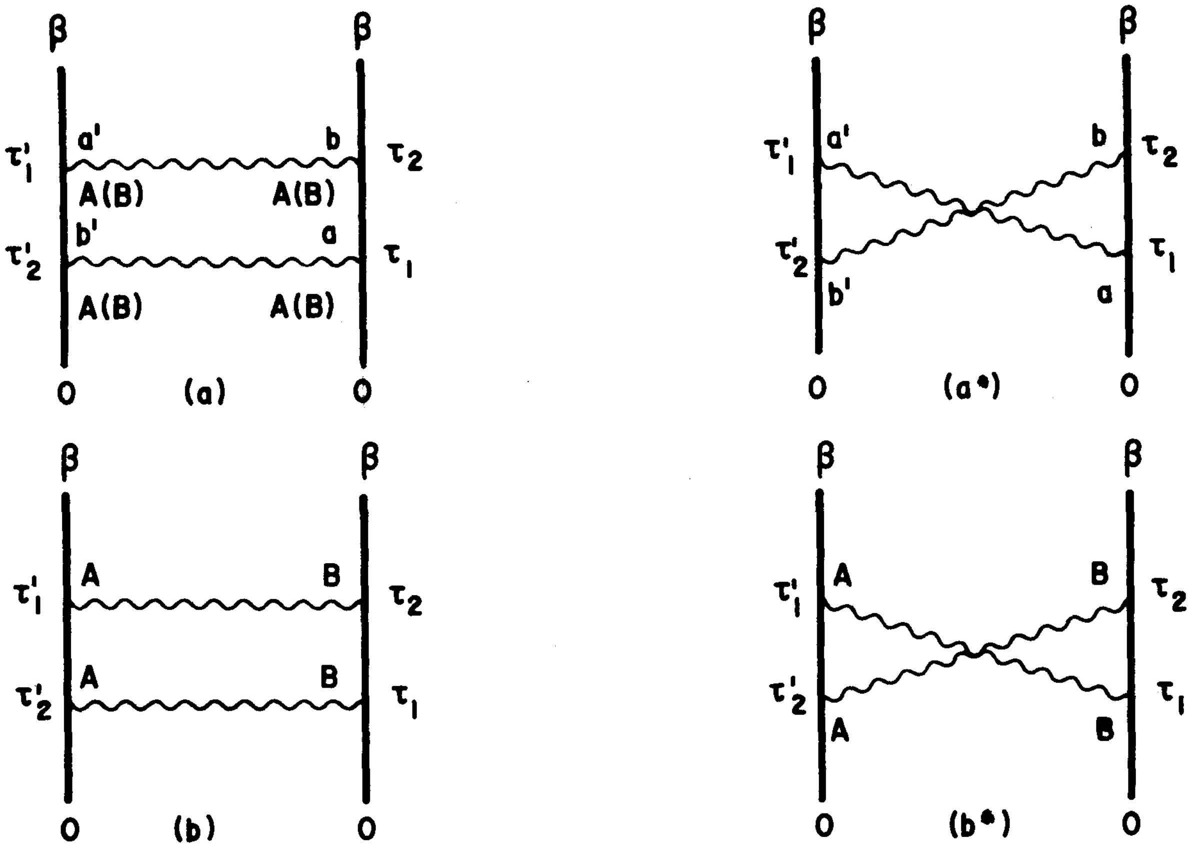

FIG.1

Diagrams contributing to the connected 2-point function of the thermal Wilson loop to order g^4. The wavy lines represent the gauge field propagators. The solid lines represent the time axis. The mixed proagator A~~~~B contributes with the graphs (b) and (b*).

By using result

$$\int_0^\beta d\tau_1'\int_0^{\tau_1'} d\tau_2'\int_\beta^0 d\tau_1\int_0^{\tau_1} d\tau_2\left[e^{i\frac{2\pi}{\beta}(n\tau_1'-n\tau_2+m\tau_2'-m\tau_1)}+e^{i\frac{2\pi}{\beta}(n\tau_1'-n\tau_1+m\tau_2'-m\tau_2')}\right]$$

$$= \begin{cases} \frac{\beta^4}{2}, & \text{for } n=0,\ m=0, \\ 0, & \text{for other cases,} \end{cases} \qquad (13)$$

one can write that

$$G_{1(A\to B)}(\vec{p}) = \frac{N^*\beta^4}{2M^4}\int d^3\vec{k}\left[\frac{1}{\vec{k}^2(\vec{p}-\vec{k})^2}-\frac{1}{\vec{k}^2[(\vec{p}-\vec{k})^2+M^2]}-\right.$$

$$\left.-\frac{1}{(\vec{k}^2+M^2).(\vec{p}-\vec{k})^2}+\frac{1}{(\vec{k}^2+M^2)[(\vec{p}-\vec{k})^2+M^2]}\right] \qquad (14)$$

which finally yields

$$G_{1(B\to A)}(\vec{x}-\vec{y}) = \frac{g^4(N^2-1)\beta^2}{2^9\pi^2N^2}\cdot\frac{[1-e^{-M|\vec{x}-\vec{y}|}]^2}{(\vec{x}-\vec{y})^2} \qquad (15)$$

and

$$G_{1(B\to A)}(\vec{x}-\vec{y}) = G_{1(A\to B)}(\vec{x}-\vec{y}) \qquad (16)$$

In a very similar way, we have evaluated the contributions of the diagrams of Fig. (1a) and Fig. (1a*). The results we have got are the following:

$$G_{1(A\to A)}(\vec{x}-\vec{y}) = \frac{g^4(N^2-1)\beta^2}{2^9\pi^2N^2}\cdot\frac{[1+e^{-M|\vec{x}-\vec{y}|}]^2}{(\vec{x}-\vec{y})^2}$$

and

$$G_{1(B\to B)}(\vec{x}-\vec{y}) = G_{1(A\to A)}(\vec{x}-\vec{y})$$

3 CONCLUSIONS

The meaning of colour confinement is a trend for particle physics. A natural way to get some understanding of it would be by looking for regions in nature where deconfinement may occur. IN QCD, this feature appears at high temperature or for high densities [5]. This means that at early stages of the universe evolution or in the interior of neutron stars quarks could be deconfined.

We have been considering a proposition to approach colour different from that of QCD. It is however also based on the existence of three colours. The motivation of this paper was the study of the behaviour of colour confinement at high temperature in this alternative framework. This new programme can be realized through two different classes of models, according to our choice of building blocks. The dynamics in this case is realized through Lagrangians that involve two gauge fields in the same group. One possibility is to consider quarks as the elementary colourful structures, but different quark families would be associated with different gauge bosons. In another model, quarks and leptons are composed by more fundamental structures referred to as colorful stones [3,4]. These stones can be of both types, bosonic and fermionic. Each family of different statistics is associated to a different gauge fields.

The method employed to calculate the interquark potential was the thermal Wilson loop. Lagrangian (3) yields three types of potentials, describing the interaction between particles of the same family, $F_{c_A \bar{c}_A}$ and $F_{c_B \bar{c}_B}$, or between particles of different families, $F_{c_A \bar{c}_B}$. Eqs. (6) and (16) yield

$$V_{A\to A}(\vec{x}-\vec{y}) = V_{B\to B}(\vec{x}-\vec{y}) = -g^4.K.\frac{[1+e^{-M|\vec{x}-\vec{y}|}]^2}{(\vec{x}-\vec{y})^2}\beta \tag{19}$$

and

$$V_{A\to B}(\vec{x}-\vec{y}) = -g^4K.\frac{[1-e^{-M|\vec{x}-\vec{y}|}]^2}{(\vec{x}-\vec{y})^2}\beta \tag{20}$$

where $k = \dfrac{N^2-1}{2^9\pi^2N^2}$

The interaction potentials of Eqs. (18) and (19) tend to zero for a very large distance between the interacting particles, meaning therefore deconfinement.

The model based on the twelve colorful stones is structured just over the colour. It can be thought of as a colorful big-bang. Therefore, it becomes interesting to investigate what happens at high temperature. Basing the dynamics of such a model on the Lagrangian of Eq. (3), what one gets is that at high temperature the colorful stones are deconfined. One could then immediately argue with which colour the deconfined structures would appear. The answer cannot be given through the thermal Wilson loop method, since it yields a colourless potential.

In experimental terms, deconfinement of the colorful building blocks could mean the possibility of measuring colour. However, we should say that we are not completely sure whether such a measurement is compatible with the quantum interpretation of the theory. The reason being that the colour charges, which appear to define superselection rules, do not commute with each other due to the non-Abelian structure of the gauge group. In view of that, it would perhaps be more sensible to ask, first of all, whether or not the colour charges can appear as observables of the theory. Or even more, one should firstly investigate if it is possible (without contradicting the quantum interpretation) to realize physical states with a non-vanishing value of the colour charges. This chain of arguments leads unavoidably to a more fundamental question: is the deconfinement of the colorful building blocks compatible with a quantum-mechanical interpretation of the theory?

As a final comment, we would like to point out, as it is known, that the time-component of the gauge fields develop an electric mass, m_{el}, arising from the loop corrections to the zero-zero component of the gauge boson self-energy, whereas the space components give rise to a magnetic mass, m_{mag}. However, since in this paper we perform calculations only up to the order g^4, these invariant loop corrections do not appear. Going to the next order, it would be expected to have an electric mass $m_{el} = f(m,T)$ and a magnetic mas $m_{mag} = g(m,T)$. They will bring a different feature to the Debye screening cutoff when compared to the one given in Ref. [6]. Finally, notice that if $m = 0$, we recover the result of Ref. [6] and the expression (17) vanishes.

ACKNOWLEDGMENTS

We are indebted to Prof. Dai Yuan-ben and Dr. Alexander

W. Smith for helpful discussions. This work would also like to acknowledge and be a part of the effort of the people at Trieste to make that place a Third world Oasis.

We would like to express our appreciation to the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for a Post-doctor fellowship (R.M.D.) and a (J.A.H.-N.). One of us (R.M.D) is thankful to Centro Brasileiro de Pesquisas Físicas for hospitality.

REFERENCES

[1] J.C. Collins and M.J. Perry, Phys. Lett. 34 (1975) 1353.

[2] A.D. Linde, Phys. Lett. 96B (1980) 289.

[3] R.M. Doria, "The Twelve Colorful Stones", CBPF-NF-024/83 (Brazilian Centre for Physical Research).

[4] R.M. Doria and J.A. Helayel - Neto, "The Twelve Colorful Stones as Building Blocks", CBPF-NF-046/83.
R.M. Doria, H.P. van Gersdorff and E.P. Silva, "Two Gauge Fields in the Same Gauge Group".

[5] D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.

[6] E. Gava and R. Jengo, ICTP Preprint IC/81/80;
Zhao Wanyun, Il Nuovo Cimento 76A (1983) 525.

[7] Zhao Wanyun,"QCD at Finite Temperature", Ph.D. thesis of the International School for Advanced Studies in Trieste.