ISSN 0029 - 3865

CBPF-NF-007/86

AN EFFECTIVE THEORY OF MASSIVE GAUGE BOSONS

by

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ABSTRACT

One studies in this work the coupling of a group-valued massive scalar field to a gauge field through a *symmetric* rank-2 field strength tensor. By considering energies very small compared with the mass of the scalar and invoking the decoupling theorem, one is left with a low-energy effective theory describing a dynamics of massive vector fields.

Key-words: Massive gauge bosons; Effective theory.

I. INTRODUCTION

The gauge principle has been used so far as the most systematic technique for describing the origin of the forces between elementary particles in terms of symmetries manifested in nature. It leads to the emergence of massless spin-1 bosons as the mediators of the interactions between matter fields and protects the associated vector bosons from acquiring mass. However, nature has up to now revealed the presence of just a single massless spin-1 particle: the photon, the gauge boson of the electromagnetic interaction. Thus a fundamental problem arises in the framework of gauge theories: the search for a mechanism that induces mass generation for the intermediate gauge bosons.

A first attempt in this direction was made by Stuckelberg already in 1938 (1); he tackled the problem of the spin-1 field mass through its coupling to a scalar that is absorbed as the longitudinal component and then induces a mass to the vector field. This approach failed however when applied to the case of self-interacting spin-1 fields.

Later, with the advent of the gauge principle and the appearence of the Yang-Mills theories (2), scalar fields could be introduced which interact through a gauge invariant self-coupling and trigger the mechanism of the hidden symmetry. It is the spontaneous symmetry breakdown aspect (3). Although it circumvent the problem of the gauge boson masslessness, it brings another situation for the spectrum of physical particles: the Higgs scalars. Despite the accurate knowledge of bounds in mass, life-time, decay properties and experimental developments, there has not yet been found any track of those fundamental particles.

Therefore, this context perfectly justifies the attitude of studying other alternative approaches for the gauge boson mass generation. Our attempt in this work follows Stuckelberg and Higgs basic idea, that is, to build up an engineering for massive vector fields through scalar particles. However it requires a change of point of view in interpreting the theory of the interacting massive gauge bosons. It will appear not as the fundamental theory built up directly from the gauge principle, but as the low-energy limit effective theory coming from the former whenever some of the heavy fields decouple. Then, massive gauge fields will be seen as a low-energy manifestation of a more fundamental theory.

The context we are trying to develop, namely, to think of a theory with massive gauge bosons as an effective low-energy limit of a symmetric and complete theory, should not be viewed as a so-surprising proposal. If we consider the Kaluza-Klein scenario, we can conclude that the usual 4-dimensional gauge theories are low-energy limits of more complete (high energies of the order of the compactification scale excite towers of massive states) and more symmetric underlying theories.

This work is outlined as follows. In Section 2, we build up the gauge action with vector and scalar fields. A symmetric "field strength" tensor is introduced. In Section 3, the low-energy effective action is studied. In the conclusion some comments about the results are presented.

2. THE GAUGE-INVARIANT ACTION

Consider the gauge covariant derivative,

$$D_\mu = \partial_\mu + ig\, A_\mu \tag{1}$$

with the potential field A_μ transforming according to

$$A'_\mu = UA_\mu U^{-1} - \frac{i}{g}\, U\, \partial_\mu U^{-1} \tag{2}$$

It yields the tensors

$$F_{\mu\nu} = [D_\mu, D_\nu] \tag{3}$$

and

$$S_{\mu\nu} = \{D_\mu, D_\nu\} \tag{4}$$

which exhibit the following transformation laws

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1} \tag{5}$$

$$S'_{\mu\nu} = U S_{\mu\nu} U^{-1} \tag{6}$$

The symmetric "field-strength" tensor has the following explicit expression:

$$S_{\mu\nu} = 2\partial_\mu\partial_\nu + 2ig(A_\mu\partial_\nu + A_\nu\partial_\mu) +$$

$$+ ig(\partial_\mu A_\nu + \partial_\nu A_\mu) - g^2\{A_\mu, A_\nu\} \tag{7}$$

Notice however that $S_{\mu\nu}$ does not belong to the Lie algebra of the gauge group as it is the case for $F_{\mu\nu}$, the reason being that the operator $\partial_\mu\partial_\nu$ appearing in (7) naturally carries the identity element. Moreover the reader should be worried for the fact that (6) is not a purely multiplicative transformation law as it is the case for (5): $S_{\mu\nu}$ carries differential operators which are supposed to act on everything appearing in the right.

(1) and (7) also produce a counter-part of the usual Bianchi identity,

$$[D_\mu, S_{\nu\rho}] + [D_\rho, S_{\mu\nu}] + [D_\nu, S_{\rho\mu}] = 0 \quad . \tag{8}$$

Having already defined the gauge covariant derivatives, let us now introduce a dimensionless scalar field transforming according to

$$\phi \rightarrow U\phi U^{-1} \tag{9}$$

and then study the transformation laws of the following quantities: $(D_\mu\phi)$, $(D_\mu D_\nu\phi)$, $(S_{\mu\nu}\phi)$ and $(F_{\mu\nu}\phi)$.

The covariant derivative D_μ given at the beginning of this section is not compatible with the transformation law proposed for ϕ. As it can be readily checked, a term like $U\phi(\partial_\mu U^{-1})$ re mains after a gauge transformation and $(D_\mu\phi)$ does not transform covariantly. Covariance for this quantity is achieved if

$$D_\mu\phi \equiv \partial_\mu\phi + ig[A_\mu,\phi] \tag{10}$$

This definition yields the following transformation law

$$(D_\mu\phi) \rightarrow U(D_\mu\phi)U^{-1} \tag{11}$$

and the strength tensors transforming like

$$(F_{\mu\nu}\phi) \rightarrow U(F_{\mu\nu}\phi)U^{-1} \tag{12}$$

$$(S_{\mu\nu}\phi) \rightarrow U(S_{\mu\nu}\phi)U^{-1} \tag{13}$$

This increases the possibilities of writing down gauge-invariant terms coupling ϕ to the gauge fields. They are

$$\mathrm{tr}\,[(S_{\mu\nu}\phi)(S^{\mu\nu}\phi)] \quad , \quad \mathrm{tr}\,[\phi S_{\mu\nu}S^{\mu\nu}\phi]$$

$$\mathrm{tr}\,[S_{\mu\nu}(\phi S^{\mu\nu}\phi)] \quad , \quad \mathrm{tr}\,[S_{\mu\nu}(S^{\mu\nu}\phi)] \tag{14}$$

Similar expressions can be obtained with the $F_{\mu\nu}$ tensor. Depending on the considerations, ϕ can be assumed to be a complex field. Another possibility that arises is to introduce gauge-invariant terms with an explicit mass parameter, such as

$$m^2[\phi D_\mu D^\mu\phi] \tag{15}$$

and

$$m^2\left[(D_\mu\phi)\,(D^\mu\phi)\right] \tag{16}$$

A final comment regards the nature of the field ϕ. The transformation law (9) opens two possibilities for ϕ: either it transforms as a number of the adjoint representation of G or it is a group element. These two possibilities have completely different characters. Being a number of the adjoint representation of G, ϕ carries a linear representation of G, but if ϕ is itself a group element, it has a non-linear expression in terms of fields which transform in the adjoint representation,

$$\phi = e^{i\alpha(x)} \quad , \quad \alpha(x) \equiv \alpha^a t_a \tag{17}$$

where α^a are real scalar fields. Substituing (17) in (9) it gives,

$$\alpha \to U\alpha\, U^{-1} \tag{18}$$

Observe that ϕ as given in (17) is *not* a vector of the adjoint representation of G.

It is however worthwhile to remark on the fact that either the transformation law (9) for ϕ nor the way the covariant derivative (10) acts on it, is sufficient to fix any particular representation of G to which ϕ may eventually belong. Eq.(9) does not select the adjoint representation of G, but if ϕ is a group element, it holds true for any particular representation of G we choose. In equation (10) we have for (17) that

$$\partial_\mu e^\alpha = e^\alpha((\partial_\mu \alpha) + \frac{1}{2}[\partial_\mu \alpha, \alpha] + \frac{1}{6}[[\partial_\mu \alpha, \alpha], \alpha] + \ldots) \qquad (19)$$

The term $[A_\mu, \phi]$ in (10) is not to be understood in the sense of the algebra, but as a commutator between the matrices corresponding to A_μ and ϕ in a given representation of G.

However, if we decide by choosing only those terms which do not contain high derivatives in ϕ and keeping in mind the requirement of renormalizability, the most immediate gauge invariant action is given by

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2\,\mathrm{tr}[\phi^+(S^\mu_\mu\phi) + (S^\mu_\mu\phi)^+\phi]$$

$$- \frac{1}{2}\mu^4\,\mathrm{tr}(\phi\phi + \phi^+\phi^+) \qquad (20)$$

where Tr denotes the trace in the adjoint representation and tr the trace in the arbitrary representation of G in which we choose to represent ϕ as a matrix.

A first motivation to take ϕ as a group element is dictated by the appearance of the term $m^2\ \mathrm{tr}(A_\mu A^\mu \phi^+\phi)$ in (20). It shows that a kind of mass term is generated already at the tree-level if ϕ is written in a given unitary representation of G. On the other hand if ϕ is not a group element but is taken to be a vector of the adjoint representation of G, the same mass term can be obtained but now at the expenses of a constraint, $\phi^{a*}\phi_a = 1$ which then breaks renormalizability.

Notice the presence of a mass term for the scalar ϕ: it is perfectly allowed by gauge invariance and will be indeed crucial for our arguments of decoupling the group-valued scalar field from the gauge fields, to get an effective low-energy theory with massive vector bosons. This is what we are going to discuss in the next section.

3 THE LOW-ENERGY EFFECTIVE THEORY

In this section, we wish to derive the low-energy effective action following from (20) after the group valued scalar field ϕ decouples. This field is redefined as $m\phi \equiv \Phi$ Φ order to have the correct canonical dimension.

We start by defining the generating functional of the full Green's functions by

$$Z[J_\mu,J] = N \int \mathcal{D}A_\mu \mathcal{D}\Phi \; e^{iS[A^\mu, \; ;J_\mu,J]} \qquad (21)$$

where the measure is gauge invariant and the integration over the group elements representd by Φ plays an important role. n

First of all, we shall discus our point of view. The scalar field has a mass that we assume to be very large, that is, it is supposed to be a field which decouples from the low-lying states of the physical particle spectrum. We then wish to study the effective "light" theory which follows from decoupling the heavy particle Φ from the original theory. By this, we mean that we are working in an energy regime such that Φ is never excited as an initial or final state or that all 1-P.I Green's functions of the gauge field exhibitin Φ's in the internal lines are supressed by powers of $(1/\mu)$ with respect to those with only gauge fields circulating in the loops [4] This effectively means that Φ is not relevant for the dynamics of the low-energy world.

In a path integral formulation, this would correspond to an integration over the field Φ. Since the contribution of this field to the classical action is quadratic and of the form

$$S = \int d^4x \left[\bar{\Phi}^{+}_{ij} M_{ik,j\ell}\, \Phi_{\ell k} + \frac{\mu^4}{m^2}\, \Phi_{ij}\Phi_{ji} + \text{h.c} \right] \tag{22}$$

(recall that being a group element, Φ is to have a matrix rep resentation, so that i,j mean indices of an arbitrary representation of the gauge group). In the above expression, the operator M is given by:

$$M_{ik,j\ell} = 2\,\Box\,\delta_{kj}\delta_{\ell i} + 4ig\, A^{\mu}_{jk}\,\delta_{i\ell}\partial_{\mu} +$$

$$- 4ig\, A^{\mu}_{\ell i}\,\delta_{jk}\partial_{\mu} + A^{\mu}_{ik}A_{\mu,\ell i} \tag{23}$$

Performing the Gaussian integration over Φ, one obtains a contribution of the form $\mathrm{tr}\,\ell n\, M_{ik,j\ell}$. So, the effective action one gets after integrating over Φ reads

$$\Gamma_{eff}[A_{\mu}] = \Gamma_{gauge} + m^2 A_{\mu}A^{\mu} - \mathrm{tr}\,\ell n\, M \tag{24}$$

To get the full local contribution to Γ_{eff}, one has to perform a perturbative expansion of the $\mathrm{tr}\,\ell n\, M$. We do it at one-loop and we have in mind that the local terms arising in this expansion can be read off from the 1-P.I graphs which are superficially divergent according to the power-counting of the theory. In our case, there are graphs which have at most four external vector lines. They are schematically shown in Fig. 1.

By drawing all graphs with ϕ internal lines which contribute to the above Green's functions, and evaluating them with the help of the Feynman rules of the theory, one gets the following kinds of local terms,

$$F^{\mu\nu}F_{\mu\nu},\ A^{\mu}\Box A_{\mu},\ f(m^2)A^{\mu}A_{\mu},\ F^{\mu\nu}[A_{\mu},A_{\nu}],\ [A^{\mu},A^{\nu}]^2 \qquad (25)$$

So that $\Gamma_{eff}[A_{\mu}]$ reads

$$\Gamma_{eff} = \Gamma_{gauge} + m^2A_{\mu}A^{\mu} + c_1F^{\mu\nu}F_{\mu} + c_2A^{\mu}\Box A_{\mu} + c_3m^2A_{\mu}A^{\mu} + $$
$$+ c_4F^{\mu\nu}[A_{\mu},A_{\nu}] + c_5[A_{\mu},A_{\nu}]^2 \qquad (26)$$

where the coefficients c_1,c_2,c_3,c_4,c_5 are to be fixed after one

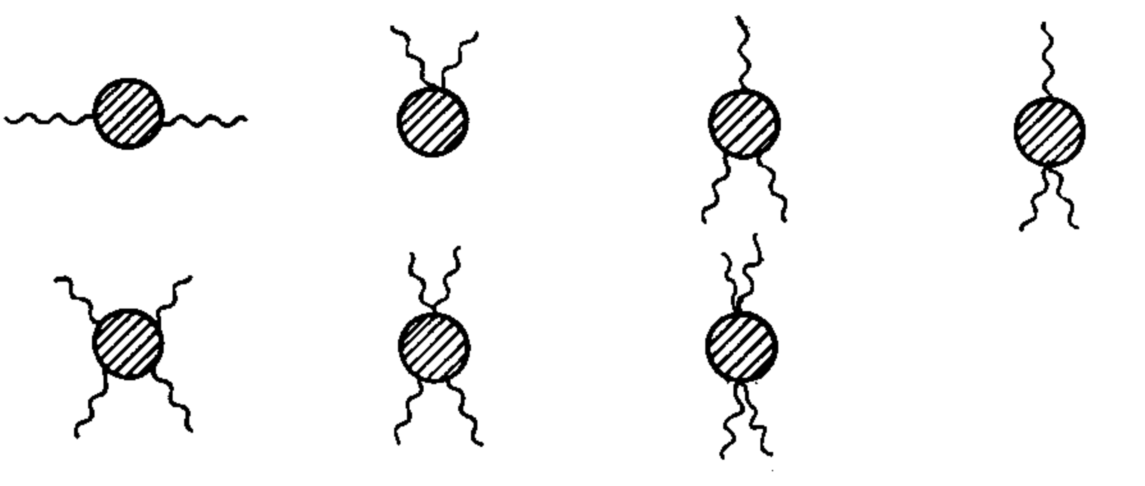

Fig. 1
Types of graphs which have a superficial divergence

regularizes these graphs (the regularization scheme adopted here is the dimensional regularization procedure [5]). The coefficients c_1 and c_3 are logarithmically divergent and contribute to the renormalization of the wave function and mass parameter of the effective theory. The coefficients c_2, c_4 and c_5 are finite and come as a direct result of the loop effects of Φ.

The remarkable feature here is to see the emergence of a mass term for the gauge field of the low-energy regime. The origin of this mass, which breaks the initial gauge invariance, can be traced back to the integration over the group elements through the functional integration over Φ. Physically, the mass generation for the gauge field can be understood through the loop effect of Φ, whose degrees of freedom are eaten up by A^μ. From this mechanism the massive vector boson gets the correct number of degrees of freedom (Φ has as many internal components as A_μ). The idea is to understand the field parameters $\alpha^a(x)$ in (17) as playing for A_μ a similar role of the Goldstone bosons.

4 CONCLUSION

In this work, we have investigated the possibility of mass generation for gauge fields through their coupling to a group-valued scalar field. This field may be given a gauge - invariant mass term and by considering the limit it becomes very heavy, we obtain a low-energy effective theory for the gauge fields after the massive scalar decouples from the system. The effective theory has the remarkable feature of describing *massive* gauge bosons whose origin may be traced back to the integration over the scalar taking values on the gauge group. This lead to an alternative way of treating massive gauge bosons: they appear as the quanta of the interactions governing a low-energy theory originating from a more complete gauge-invariant theory dominating at a very high energy regime. Our point of view is that the massive gauge bosons manifest themselves at low energies (the ones we are able to test) as the residual quanta of a full theory whose heavy sector has decoupled.

In this framework the first consideration is about a massive QCD - like theory. The fact that a single quark has never been observed brings a special situation for a theory based on colour. However despite the absence of a direct experimental observation there are different objective arguments to consider quarks with three degrees of freedom. This context makes the basis for the gauge technique to assume the $SU(3)_c$ group. It yields QCD. Our observation is that such a situation is not against the presence of massive gluons. QCD thesis must be interpreted just as a first stage for comparing with QED theory. For instance, the electric charge yields the Coulomb' law in QED. However applying the same procedure in QCD yields that the chromostatic force is not able to justify the confinement.

It is not sensible to impose arguments about colour matter without bearing in mind its experimental difficulties. $SU(3)_c$ is the only principle behind. *Therefore it turns necessary to have a co-existence with the different departures* *from this principle.* Our attitude is that before to look for new theoretical methods as dividing the space-time continuum into a lattice of discrete points, *we should first ask about gluons experimental conclusions.* There are good reasons to assume that gluons are vectors (6) . However the question about their masses is open.

There is no precise experimental background for discussion about gluon masses. Our point of view will be developed with three qualitative considerations. They are: an explanation for the difference between current and constituant mass for quarks, to think about the meaning that half of the total proton momentum is thought to be carried by weak and electrically neutral particles and to interpret glueballs. For the first consideration massive gluons can appear as an affirmation of the eightfoldway model. This means that the eightfoldway orthodoxy would be preserved if massive gluons assume the difference of mass that characterizes the current quarks. The second aspect is from the momentum sum rules in electron, muon and neutrino-photon scattering. However since the association between momenta and mass is not direct, this experimental fact does not reveal if gluons are massive or not. The most important proof will be if glueballs are massive or not. The possibility of their masses to be around thousand times bigger than the electron mass suggests the massive gluon thesis.

Theoretically a massless gauge boson yields infrared problems in QCD for a colourful initial state (7). An explanation for the break of factorization theorems in the quark-gluon process (by using dimensional regularization) is still open. Thus a massive vector field would be welcome for a theoretical consistency of non-abelian gauge theories.

Neverthless symmetry dictates just massless gauge fields. the gauge freedom indeed works in order to obtain the correct number of degrees of freedom. This limitation has motivated to develop a theoretical laboratory involving more degrees of freedom. Thus Stuckelberg, Higgs and others created a framework with scalar fields. Following this behaviour the effort in this text was to develop an effective theory. (26) *yields a new vector field to communicate the local colour conventions from place to place.* It carries not only colour internal indices but also mass. Notice that a more general case can be studied if we consider a more general gauge invariant action by including all the terms listed in (14).

Qualitatively we could expect that an effective massive QCD will preserve the standard QCD properties. Gluon shielding and the assymptotic freedom property [8] are expected to be obtained. A problem for massive gluons obtained via Higgs particles was the possible loss of the assymptotic freedom behaviour. Calculations under study expect to show that a non-abelian effective theory given by (26) yields a free particle behaviour at high energy. Although it is a non-simple task it is also expected a behaviour of the structure function being not inconsistent with the prediction of an effective massive QCD. Scaling violations can also support such a model. Other phenomena in hadron-hadron collisions are jets. Considering the kinematical relevance the jet production mechanism is not expected to change qualitatively due to the exchange of a single massive gluon between two passing quarks. However any effective model from

(14) or (26) yields just a static potential of the Yukawa type. Our consideration is that a hint for confinement would be to obtain a linear potential at the three level. Therefore, this model fails to investigate quarks with confinement. However the phenomenology of weak interactions is offering us motivation for studying the application of this mechanism of vector boson mass generation.

Finally we would like to observe the commom sense where interactions should be dictated by symmetry loses its generalization. The non-gauge invariant interaction that is generated in the low energy effective theory of this work is just a trace of a gauge symmetry at high energy. This means that *symmetry only organizes the degrees of freedom as its most primitive physical entity*. Therefore the mass term at low energy conserves the initial degrees of freedom. However its archeology can be found in the interactions at high energy that have disappeared in the decoupling process. Nevertheless an interesting aspect here is that, the break of gauge invariance that the appearance of a mass term shows, can be interpreted similarly ot the Faddev-Popov case (9) . It is in the sense that the integration over the group element have also works as a kind of measure over gauge equivalent configurations.

ACKNOWLEDGEMENTS

The authors wish to thank K. Fujikawa for useful discussions.

We would like also to express our gratitude to Coca-cola of Brazil, through Sonia Barreto, and to Johnson & Higgins, through Dr. Michael Wyles, for the invaluable financial help.

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