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A NEW GAUGE FOR SUPERSYMMETRIC ABELIAN GAUGE THEORIES

by

Alexander W. Smith and J. Barcelos Neto[§]

Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ, Brasil

[§]Instituto de Física - UFRJ
Cidade Universitária
Ilha do Fundão
21944 - Rio de Janeiro, RJ, Brasil

Abstract

A new gauge for supersymmetric abelian gauge theories is presented. It is shown that this new gauge allows us to obtain terms which usually come as radiative corrections to the supersymmetric abelian gauge theories when one uses the Wess-Zumino gauge.

Key-words: Supersymmetric abelian gauge theories.

1. Introduction

It has already been pointed out the importance of dealing with superfields [1] i.e., that of working with linear representations of supersymmetry. But we face the problem of having many degrees of freedom corresponding to non physical fields. So in this way one usually tries to eliminate some of them in the following ways: (i) by imposing convenient constraints on general superfields and the most common ones are

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \tag{1.1}$$

$$D_{\alpha}\Phi^{+} = 0 \tag{1.2}$$

$$V^{+} = V \tag{1.3}$$

where D_{α} and $\bar{D}_{\dot{\alpha}}$ are the usual covariant derivatives given by

$$D_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} + i\,\sigma^{m}_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\frac{\partial}{\partial x^{m}} \tag{1.4a}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\theta^{\dot{\alpha}}} - i\,\theta^{\beta}\,\sigma^{m}_{\beta\dot{\alpha}}\,\frac{\partial}{\partial x^{m}} \tag{1.4b}$$

Φ, Φ^{+} and V are called chiral, anti-chiral and vector superfields, and they are used to construct supersymmetric gauge invariant actions [2,3]. These superfields are given in terms of component fields by

$$\Phi(x,\theta,\bar{\theta}) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x) +$$

$$+ i\theta\sigma^m\bar{\theta}\partial_m A(x) + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^m\partial_m\psi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\Box A(x) \qquad (1.5)$$

$$\Phi^+(x,\theta,\bar{\theta}) = A^*(x) + \sqrt{2}\,\bar{\theta}\bar{\psi}(x) + \bar{\theta}\bar{\theta}F^*(x)$$

$$-i\theta\sigma^m\bar{\theta}\partial_m A^*(x) + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^m\partial_m\bar{\psi}(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\Box A^*(x) \qquad (1.6)$$

$$V(x,\theta,\bar{\theta}) = C(x) + \theta\chi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) +$$

$$+ \bar{\theta}\bar{\theta}M^*(x) + \theta\sigma^m\bar{\theta}v_m(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \qquad (1.7)$$

In the last expression, $C(x)$, $v_m(x)$ and $D(x)$ are real fields. We are using the same notations and conventions of reference [2]. (ii) component fields of too high a dimension normally enter the action without any derivatives (auxiliary fields) and can be eliminate from the action by using their Euler-Lagrange equation. (iii) requiring the theory to be gauge invariant, we can eliminate some component fields from V through the supersymmetric gauge transformation given by

$$V' = V + \Lambda + \Lambda^+ \qquad (1.8)$$

where Λ and Λ^+ are chiral and anti-chiral superfields.

The degrees of freedom of the superfields Λ and Λ^+ are used to eliminate the component fields $C(x)$, $\chi(x)$ and $M(x)$, which have (mass) dimensions zero, 1/2 and 1, respectively from the superfield V. This particular gauge is known in the literature as the Wess-Zumino (WZ) gauge. After using this gauge, the vector superfield is given by

$$V(x,\theta,\bar{\theta}) = i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) - \theta\sigma^m\bar{\theta}v_m(x) + \frac{1}{2}\theta^2\bar{\theta}^2D(x) \tag{1.9}$$

where λ has dimension $\frac{3}{2}$ and is the supersymmetric partner of the field $v_m(x)$ which has dimension 1 and is the vector gauge field. $D(x)$ is an auxiliary field and has dimension 2.

The purpose of this work is to present a gauge, different from the WZ one, and analyse the new couplings which will appear in developing the supersymmetric action.

2. A new gauge

We are also going to use the gauge transformation given by (1,8) but now with the aim of gauging away the fields $\lambda(x)$ instead of $\chi(x)$ and also as before, $M(x)$ and $N(x)$.

Then one reads for the superfield V:

$$V(x,\theta,\bar{\theta}) = \theta\chi(x) + \bar{\theta}\bar{\chi}(x) - \theta\sigma^m\bar{\theta}v_m(x) + \frac{1}{2}\theta^2\bar{\theta}^2D(x) \tag{2.1}$$

At this stage one should be worried since we have just gauged away the degrees of freedom of the would be supersymmetric partner of the vector field (the photino for supersymmetric QED). But as we will see later this problem can be circumvented by a suitable redefinition of the component field $\chi_\alpha(x)$. With this new gauge we have $V^n(x,\theta,\bar\theta) = 0$ for $n \geq 5$, differently from the WZ one where we had $V^n(x,\theta,\bar\theta) = 0$ for $n \geq 3$. This new gauge allow us to obtain directly g^3 and g^4 couplings without going to high order corrections, as we have to do in the case of the WZ gauge.

Aparently we have a trouble with this new gauge, since χ has dimension 1/2 and cannot be associated to a physical field. This problem is contorned by redefining conveniently this component field in the following way:

$$\chi_\alpha \to -\frac{2}{\Box} \sigma^m_{\alpha\dot\beta}\partial_m \bar\chi^{\dot\beta} \tag{2.2}$$

$$\bar\chi^{\dot\alpha} \to \frac{2}{\Box} \bar\sigma^{m\dot\alpha\beta} \partial_m \chi_\beta \tag{2.3}$$

So the new field χ has dimension 3/2, as it must be. With the replacements (2.2) and (2.3) we have

$$V'(x,\theta,\bar\theta) = -2\theta\sigma^m \frac{\partial_m}{\Box} \bar\chi + 2\bar\theta\bar\sigma^m \frac{\partial_m}{\Box} \chi -$$

$$- \theta\sigma^m\bar\theta v_m(x) + \frac{1}{2} \theta^2\bar\theta^2 D(x) \tag{2.4}$$

Let us now analyse this new gauge in the supersymmetric gauge invariant and renormalizable action, which is given by

$$I = \int d^8z [\Phi^+ e^{gV}\Phi + \frac{1}{4} W^\alpha W_\alpha \delta(\bar\theta) + \frac{1}{4} \bar W_{\dot\alpha} \bar W^{\dot\alpha} \delta(\theta)] \tag{2.5}$$

where $$d^8z \equiv d^4x\, d^2\theta\, d^2\bar\theta$$

In the above action we did not write terms like $m^2\phi^2$ and

$g'\Phi^3$ since they are not affected by this new gauge choice. The last two terms are the kinetic ones and W_α, $\bar{W}_{\dot\alpha}$, $\delta(\theta)$ and $\delta(\bar\theta)$ are defined by

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V \tag{2.6}$$

$$\bar{W}_{\dot\alpha} = -\frac{1}{4}DD\bar{D}_{\dot\alpha}V \tag{2.7}$$

$$\delta(\theta) = \theta^\alpha\theta_\alpha \tag{2.8}$$

$$\delta(\bar\theta) = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha} \tag{2.9}$$

Replacing $V(x,\theta,\bar\theta)$ which appears in (2.6) and (2.7) by (2.4) we obtain for the kinetic part of action (2.5):

$$I_{kin} = \int d^4x\,(-\frac{1}{4}F_{\ell m}F^{\ell m} + \frac{1}{2}D^2 - i\chi\sigma^m\partial_m\bar\chi) \tag{2.10}$$

where $F_{\ell m}$ is the known antisymmetric tensor for the abelian case, whose definition is

$$F_{\ell m} = \partial_\ell v_m - \partial_m v_\ell \tag{2.11}$$

As we observe, the kinetic part of the action is the same of that obtained if we had used the WZ gauge (see ref. [2]). With regard to the first term of action (2.5), we have:

$$\Phi^+ e^{gV}\Phi = \Phi^+\Phi + g\Phi V\Phi + \frac{g^2}{2!}\Phi^+V^2\Phi + \frac{g^3}{3!}\Phi^+V^3\Phi + \frac{g^4}{4!}\Phi^+V^4\Phi \tag{2.12}$$

Using (1.5), (1.6) and (2.4) we obtain:

$$I_o = \int d^8z\, \Phi^+\Phi = \int d^4x [A^* \Box A + FF^* - i\psi\sigma^m\partial_m\psi] \tag{2.13}$$

$$I_1 = g\int d^8z\, \Phi^+V\Phi = g\int d^4x [\frac{i}{2} v^m A^* \partial_m A - \frac{i}{2} v^m A\partial_m A^* +$$

$$+ \frac{i}{\sqrt{2}} A^*\chi\psi - \frac{i}{\sqrt{2}} A\bar{\chi}\bar{\psi} + \frac{1}{2} AA^*D - \frac{1}{2} v_m\psi\sigma^m\bar{\psi} + \sqrt{2}\, F \frac{\partial_m}{\Box} \chi\sigma^m\bar{\psi} +$$

$$+ \sqrt{2}\, F^*\psi\sigma^m \frac{\partial_m}{\Box} \bar{\chi} - i\sqrt{2}\, \partial_m A^*\psi\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi + i\sqrt{2}\, \partial_m A \frac{\partial_\ell}{\Box} \bar{\chi}\bar{\sigma}^\ell\sigma^m\bar{\psi}]$$

(2.14)

$$I_2 = \frac{g^2}{2} \int d^8z\, \Phi^+V^2\Phi = \frac{g^2}{2} \int d^4x [-\frac{1}{2} A^*A\, v_m v^m +$$

$$+ 2A^*F \frac{\partial_m}{\Box} \chi\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi + 2AF^* \frac{\partial_m}{\Box} \bar{\chi}\bar{\sigma}^m\sigma^\ell \frac{\partial_\ell}{\Box} \bar{\chi} +$$

$$+ 2i(A^*\partial_m A - A\partial_m A^*) \frac{\partial_n}{\Box} \bar{\chi}\bar{\sigma}^n\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi - \sqrt{2}\, A^* v_m \psi\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi -$$

$$- \sqrt{2}\, Av_m \frac{\partial_\ell}{\Box} \bar{\chi}\bar{\sigma}^\ell\sigma^m\bar{\psi} - 4\bar{\psi}\sigma^m \frac{\partial_m}{\Box} \chi\psi\sigma^\ell \frac{\partial_\ell}{\Box} \bar{\chi}] \tag{2.15}$$

$$I_3 = \frac{g^3}{3!} \int d^8z\, \Phi^+V^3\Phi = g^3\int d^4x [\sqrt{2}\, A \frac{\partial_n}{\Box} \bar{\chi}\bar{\sigma}^n\sigma^\ell \frac{\partial_\ell}{\Box} \bar{\chi} \frac{\partial_m}{\Box} \chi\sigma^m\bar{\psi} +$$

$$+ \sqrt{2}\, A^* \frac{\partial_n}{\Box} \chi\sigma^n\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi\psi\sigma^m \frac{\partial_m}{\Box} \bar{\chi} - AA^*v_m \frac{\partial_n}{\Box} \bar{\chi}\bar{\sigma}^n\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi]$$

(2.16)

$$I_4 = \frac{g^4}{4!} \int d^8z\, \Phi^+V^4\Phi = g^4 \int d^4x\, AA^* \frac{\partial_n}{\Box} \bar{\chi}\bar{\sigma}^n\sigma^\ell \frac{\partial_\ell}{\Box} \bar{\chi} \frac{\partial_m}{\Box} \chi\sigma^m\bar{\sigma}^\ell \frac{\partial_\ell}{\Box} \chi$$

(2.17)

So

$$I = I_0 + I_1 + I_2 + I_3 + I_4 = \int d^4x \{ [FF^* + A\Box A^* - i\psi\sigma^m\partial_m\bar{\psi} +$$

$$+ gv^m(\frac{1}{2}\bar{\psi}\bar{\sigma}_m\psi + \frac{i}{2}A^*\partial_m A - \frac{i}{2}\partial_m A^* A) - \frac{ig}{\sqrt{2}}(A\bar{\chi}\bar{\psi} - A^*\chi\psi) +$$

$$+ \frac{1}{2}(gD - \frac{1}{2}g^2 v_m v^m)A^*A] + [g(\sqrt{2}F\frac{\partial_m}{\Box}\chi\sigma^m\bar{\psi} + \sqrt{2}F^*\psi\sigma^m\frac{\partial_m}{\Box}\bar{\chi} -$$

$$- i\sqrt{2}\,\partial_m A^*\psi\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi + i\sqrt{2}\,\partial_m A\frac{\partial_\ell}{\Box}\bar{\chi}\bar{\sigma}^\ell\sigma^m\bar{\psi}) +$$

$$+ \frac{g^2}{2}(2A^*F\frac{\partial_m}{\Box}\chi\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi + 2AF^*\frac{\partial_m}{\Box}\bar{\chi}\bar{\sigma}^m\sigma^\ell\frac{\partial_\ell}{\Box}\bar{\chi} +$$

$$+ 2i(A^*\partial_m A - A\partial_m A^*)\frac{\partial_n}{\Box}\bar{\chi}\bar{\sigma}^n\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi - \sqrt{2}A^*v_m\psi\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi -$$

$$- \sqrt{2}Av_m\frac{\partial_\ell}{\Box}\bar{\chi}\bar{\sigma}^\ell\sigma^m\bar{\psi} - 4\bar{\psi}\sigma^m\frac{\partial_m}{\Box}\chi\psi\sigma^\ell\frac{\partial_\ell}{\Box}\bar{\chi}) +$$

$$+ g^3(\sqrt{2}A\frac{\partial_n}{\Box}\bar{\chi}\bar{\sigma}^n\sigma^\ell\frac{\partial_\ell}{\Box}\bar{\chi}\frac{\partial^m}{\Box}\chi\sigma_m\bar{\psi} + \sqrt{2}A^*\frac{\partial_n}{\Box}\chi\sigma^n\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi\psi\sigma^m\frac{\partial_m}{\Box}\bar{\chi} -$$

$$- AA^*v_m\frac{\partial_n}{\Box}\bar{\chi}\bar{\sigma}^n\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi) + g^4(AA^*\frac{\partial_n}{\Box}\bar{\chi}\bar{\sigma}^n\sigma^\ell\frac{\partial_\ell}{\Box}\chi\frac{\partial_m}{\Box}\chi\sigma^m\bar{\sigma}^\ell\frac{\partial_\ell}{\Box}\chi)]$$

(2.18)

where in the first braket we have the usual Lagrangian of reference [2]. The terms in the second bracket constitute those which one should expect to obtain from radiative corrections to the Lagrangian of reference [2]. Here we have obtained them just by choosing a different gauge than that of Wess-Zumino.

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