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PULSAR PRECESSION: A NOD IS NOT AS GOOD AS A WINK!

by

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ABSTRACT

The problem of pulsar wobble is reasessed and the most favourable cases are selected. Chances for an actual detection appear to be dim.

INTRODUCTION

It is sometimes said, that pulsars could have been predicted before their actual discovery (in 1967) but this is certainly not the case, as evidenced by the fact, that even 15 years later the emission mechanism, i.e. the very basis for their detection, is still unknown and is likely to remain so for quite some time. As a matter of fact, the now generally accepted pulsar model (a rotating, magnetized neutron star) was developed in an astonishingly short time and has remained essentially unchanged since. To the very few predictions, which can be made on general grounds, belong the observed slowing of pulsar rotation, the unobserverd internal excitations of a neutron star, such as pulsation or torsional oscillation, and last not least precession. All these effects have by now been observed in case of the Earth.

All three effects are important indicators of the global structure of a rigid body. Precession, both free and forced, is therefore a pulsar evergreen. In fact there is every theoretical reason to believe that it should be there, yet observations put severe limits on both period P_W and amplitude θ_W of a possible wobble. We want to show here that earlier estimates were overoptimistic as far as the observability is concerned and there is as yet no danger in sight for (at least the more modern) pulsar models.

When pulsars were first discovered their masses were essentially unknown and it was natural to assume that they could vary considerably. Nowadays we do not believe that a pulsar's mass is a very important parameter, not because low mass neutron stars are forbidden to exist theoretically but because they are not formed under ordinary circumstances. The Chandrasekhar mass seems to be a lower limit for a typical neutron star and some fast pulsars may even be heavier due to subsequent accretion. With hindsight we could have predicted therefore, that any modulation of a pulsar's pulse period will be very difficult to observe indeed due to the excessively large wobble period in combination with strong timing noise.

Before we enter into a discussion about pulsars, it may be helpful to recall the situation for the Earth. Forced precession of the Earth is quantitatively accounted for by Newton's theory. The Earth is to a very good approximation a prolate top, spinning progradely in the combined gravitational fields of the moon, the sun and the planets (called unfortunately in the astronomical literature nutation and precession of the equinoxes). The Earth has a solid crust and is therefore an elastic top. As a matter of fact the crust of the Earth has the rigidity of steel and is thereby selfsupporting (in contradistinction to pulsars, where the crust is jelly). As the Earth rotated much faster at formation one could expect that its actual deformation exceeds the equilibrium value of an entirely liquid body, and due to the rather chaotic formation process (by coagulation of planetesimals) it could even by a triaxial top with principal moments of inertia $A<B<C$. This is not so as, we know from measurements of the geoid ($A=B<C$), which go back to F. Gauss. As a matter of fact, the deformation of the Earth corresponds to its fluid value

and we infer, that the Earth is plastic rather than elastic on large timescales, as evidenced directly by glacial rebound, which proceeds on timescales of million years.

From the prediction by Euler to the detection by Chandler many careful observations failed to detect the Earth's free precession partly because the effect is very small (amplitude θ_W = 0.''1) and partly because the predicted period was in error (as it was assumed that the Earth's rigidity was infinite). Kelvin explained then that the elastic yielding of the Earth would lengthen the period. From the known properties of solid matter he concluded that the rigidity of the Earth is that of steel. Actually the situation is slightly more complicated since there is a liquid interior (which moreover does not coprecess) and a liquid ocean on top. By accident both effects nearly compensate each other, but for pulsars this will not be the case. Before we turn to assess the chance to actually observe precession in pulsars one further relevant and disconcerting remark may be in order about the situation of the Earth: the Chandler-wobble is strongly damped or equivalently the quality factor Q is rather low, $Q: = P_W/\Delta P_W = 10$; and the phase shows large variations without any obvious physical reason.

Nutation and wobble: the pulsar model.

In order to be able to detect a pulsar's free precession its amplitude must be large and its frequency and quality factor high. We will assess now all three conditions in turn and in doing so we try to rely as much as possible on observations. Here observations of the Crab pulsar and of the X-ray binary Her-X1 are especially valuable (1). As the terminology in the literature is rather confusing and in order to be understood unequivocally let us repeat briefly some facts about the free motion of a spinning symmetrical top. Its motion is determined completely by the specification of the Eulerian angle variables ϕ, ψ, θ with the solution

$$\phi = \frac{S}{A}t; \quad \psi = \frac{\pi}{2} - \frac{S.(C-A)}{A.C}\cos\theta_w; \quad \theta = \theta_w \tag{1}$$

The following transformation matrix relates inertial (x, y,z) and corotating coordinates (1,2,3)

$$\begin{array}{c|ccc} & e_x & e_y & e_z \\ e_1 & \cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta & \cos\psi\sin\phi + \sin\psi\cos\phi\cos\theta & \sin\psi\sin\theta \\ e_2 & -\sin\psi\cos\phi - \cos\psi\sin\theta\cos\theta & -\sin\psi\sin\phi + \cos\psi\cos\phi\cos\psi & \cos\psi\sin\theta \\ e_3 & \sin\phi\sin\theta & -\cos\phi\sin\theta & \cos\theta \end{array} \tag{2}$$

As far as observable quantities are concerned Rüderman (2) has given the complete answer, which we repeat here in somewhat different notation. As we shall show ahead it is likely that the symmetry axis of a pulsar coincides with the direction of the magnetic field (aligned or orthogonal rotator), so we concentrate on the following two limiting cases, where the emission is either along the polar axis or along an equatorial axis, i.e. along the 3-axis and e.g. along the 1-axis of the pulsar. Obviously the center of a pulse will arrive (for a conical beam) for an observer in the x-z plane if the y-component of the corotating axis vanishes. This defines the pulsar phase and its derivative, the pulse period. We obtain for the pulse phase $\tilde{\Phi}$:

$$\tilde{\Phi} = \phi$$

and

$$\sin\tilde{\Phi} = (1 - \sin^2\psi \sin^2\theta_W)^{-1/2} . (\cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta_W) \qquad (3)$$

for the two cases respectively, in agreement with ref. (2). To first approximation we find for the pulse phase in the second case

$$\tilde{\Phi} = \phi + \psi + \frac{1}{2}(1 - \cos\theta_W)\sin 2\psi \qquad (4)$$

(The "lenght of the day" is S/A at the pole and S/C at the equator!). In order to proceed we must specify our pulsar model in more detail. There are at present two schools of thought about pulsar magnetic fields which can best be characterized by the adjectives fossil or dynamical. Here we are not primarily concerned with the question of wether pulsar magnetic fields can be generated or destroyed (3), but what their relative importance is: we must know if they permeate the whole star or if they are anchored only in the outer crust. Here only observation can help and this (of course only indirectly) as follows. If the magnetic field does not permeate the whole star, braking of the core will be effected by viscous drag and for a young neutron star with an internal temperature in excess of $10^{8\,0}$K the viscous coupling time will exceed a year (4).

$$\tau_{visc} \approx \rho R^2 \eta^{-1} \approx 10^8 s\, \rho_{15} R_6^2 \eta_{19}^{-1} \qquad (5)$$

easily detectable in the timing noise of the Crab and Her-X1. As this is not the case (1), we conclude that pulsars are not entirely field-free in their interiors, but to be fair we point out that fields of the order of 10^{10} Gauss would have an (Alfven) coupling time of hours, (4), and hence be unobservable by present pulsar timing routines. Therefore the model of a dynamical magnetic field, (3), residing in the crust is viable and leads to the standard pulsar model, where the magnetic field is inclined by an arbitrary angle to the rotation axis. In this case the symmetry axis of the rigid part of the pulsar is determined by the crust and as pulsars are born in the liquid state there is no reason to expect the pulsar phase to wobble at all (amplitude zero).

In case the magnetic field permeates the whole star the crust will play only a minor role (Crab, Vela), and in most pulsars it will be negligible. The situation is depicted in Fig. 1. In order to evaluate the relative importance of crust and magnetic field for the rigidity of the neutron star we introduce the parameter η_G, where the index G stands for Goldreich, who first analized the situation (5), and which is defined as the ratio between the respective precession periods: $\eta_G := P_W(B)/P_W(\text{crust})$.

From Pines et al. (6) we adopt $P_W(\text{crust}) = 15$ years for a neutron star of $1{,}4M_\odot$ and a period of 33ms (the Crab) and from ref. 5 we adopt $P_W(B) = 6$ years for the same object. We see that $\eta_G = 0.4$, which implies that the magnetic field and crustal rigidity are of equal importance in case of the Crab pulsar. As a matter of fact Goldreich assumed in his analysis a constant internal magnetic field, which is probably an underestimate, as is clear from the work of Mestel et. al., which treats the analogous problem for ordinary stars, (7). If neutron stars are superconducting a further enhancement of the magnetic field energy will occur, (8), and η_G will decrease further. We adopt therefore as a conservative estimate (the index C stands for Crab and $\tau := P/2\dot{P}$) $\eta_G^{-1} = B_C^2 P_C^2 = \dot{P}_C P_C^3 = B_C^4 \tau_C$, and inspection of Fig. 1 shows that only half a dozen of the 300 or so pulsars have their symmetry axis determined by the crust and in most of the pulsars the magnetic field dominates by orders of magnitude. Obviously we have arrived at an entirely new pulsar model, which is actually quite old. In fact in 1970 Axford et al. have considered already this problem (in different context but with formally similar conclusion, (9)). Hence the title of the present paper.

Let us discuss some general implications of the model first. If the magnetic field determines the symmetry of the pulsar the star must precess in order that pulses can be emitted and therefore the precession angle θ_W must be larger than the pulse width. To be exact the period of the pulses is given by the period of free nutation, which according to the above said is S/A. No wobble will be present in the pulse arrival times if the symmetry axis coincides exactly with the magnetic field. In the fast rotating pulsars however the symmetry axis will not coincide exactly with B since the contribution of the crust cannot be neglected: the best pulsars for the detection of a wobble are however those for which η_G is of order unity. There is another important consequence of this model: it explains rather naturally the absence of interpulses in long period (i.e. in most of the) pulsars, provided one accepts the following considerations: in order to see interpulses it is necessary that the magnetic field axis be inclined by more than 45 deg. w.r.t. the rotation axis. If the (poloidal) magnetic field determines the rigidity this means that as far as precession is concerned for which only the rigid part of the top counts (a liquid body cannot precess freely) we are dealing with a prolate top, i.e. a top, which is spinning about the axis of the minor moment of inertia. If the inclination-angle is not exactly 90 deg, (orthogonal rotator) such a motion is known to be unstable if the top contains some liquid part. The present model explains therefore the observations well, if we make the (ad hoc?) assumption, that internal dissipation is proportional to (some power of) the precession amplitudes θ_W. In this case any initial obliquity is rapidly damped away until damping becomes negligible on the slow-down time scale. Here we have arrived at the weakest point of the model (5,9): it predicts an unobserved alignment i.e. pulse broadening in old (slow?) pulsars, or it necessitates at least two different classes of pulsars: long period pulsars, which are old (those with broad pulses!) and long period pulsars, which are young (those with narrow pulses!). Our model is therefore independent evidence that pulsars are "injected" with long periods, (10).

What are the chances to actually detect a pulsar wobble period? In tables 1 and 2 we have summarized some relevant data. Ideally a pulsar should have a short period, an η_G of order unity (large amplitude!) and be noise free. There is no such pulsar! Crab and Vela are too noisy, as for a 3σ detection 10 wobble periods are required: at least 60 years if the noise is frequency noise and more if it is torque noise, (1)!

CONCLUSION

As first pointed out by Ruderman a possible pulsar wobble will appear as a modulation of the pulse arrival times, and under favorable circumstances its amplitude will be 1. Unless neutron stars have solid interiors the wobble periods are too long to be detectable (by a Fourier analysis) or equivalently the relevant pulsars are too noisy. It is to be feared that the recently discovered pulsar PSR 1508-58 will be no exception from this rule.

Table 1

List of pulsars with interpulses.
(The mean value for η_G^{-1} is 150)

Pulsar designation (PSR)	η_G^{-1}	Period (ms)
1937+21	$2*10^{-1}$	1,5
1944+17	0,2	440
0950+08	0,25	253
1929+10	1	226
0531+21	1	33
1055-52	3	197
0823+26	16	530

Table 2

List of pulsar wobble periods

Pulsar designation (PSR)	η_G^{-1}	P_W (years)
1508-58	0,27	4,5
0531+21	1	6
1916+14	$2*10^4$	14
0154+61	10^5	15
0833-45	7	17

FIGURE CAPTION

Fig. 1.-- Lines of constant $\eta_G = B_c^{-2} P_c^{-2}$ in a P, $\dot{P}$ diagram. Note that distortions add up like η^{-1}, which is given in tables 1 and 2 for selected pulsars.

+ pulsars with interpulses

θ Binary pulsars

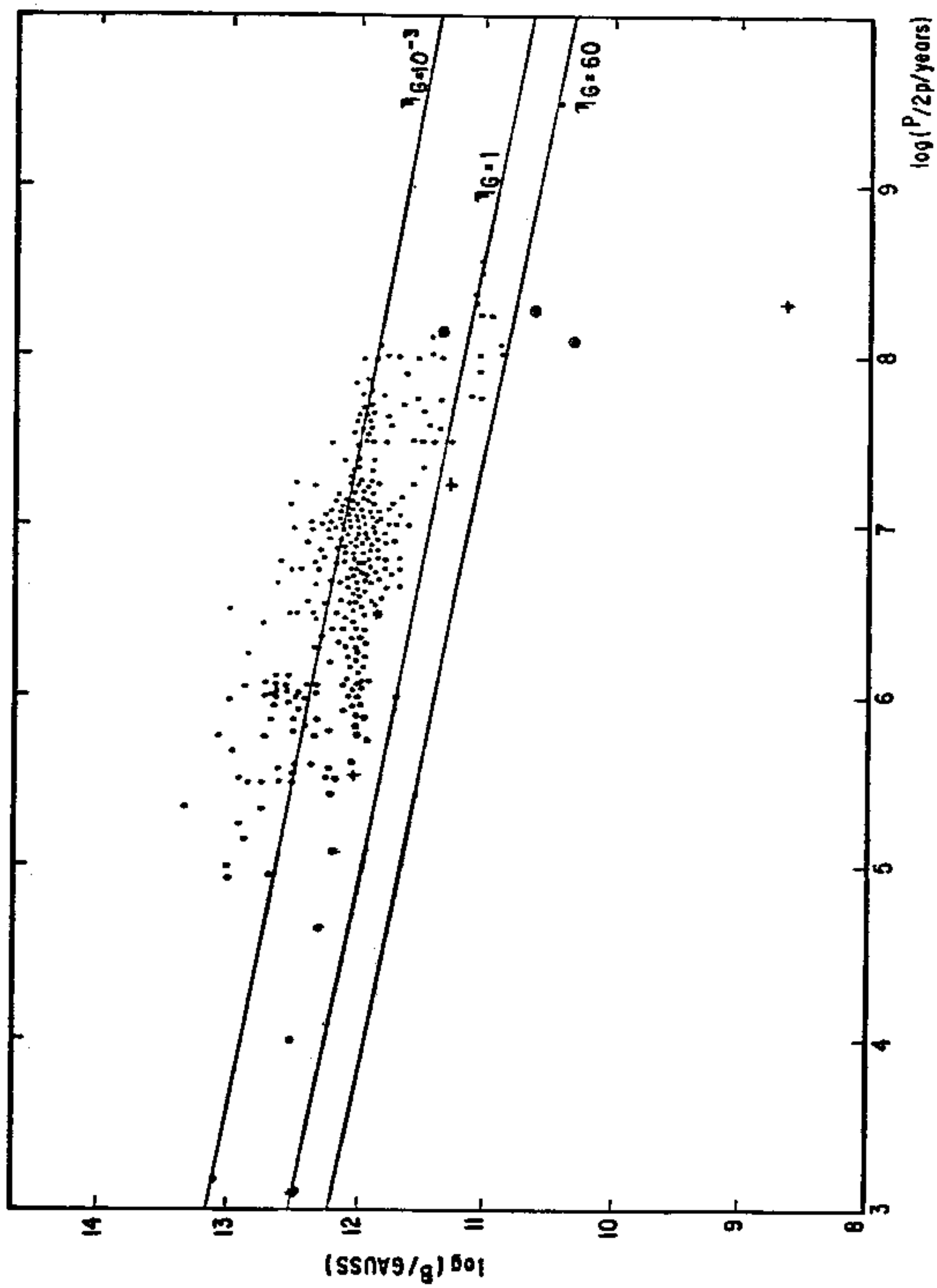
log(B/GAUSS)
log(P/2p/years)
$\eta_G = 10^{-3}$
$\eta_G = 1$
$\eta_G = 60$
14
13
12
11
10
9
8
3
4
5
6
7
8
9

Fig. 1

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