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DIFFRACTIVE DISSOCIATION IN $pp \rightarrow \Delta^{++}\pi^{-}p$.
II. SLOPE-MASS-PARTIAL WAVE CORRELATION

by

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Abstract

This paper is a continuation of the "Diffractive Dissociation in $pp\rightarrow\Delta^{++}\pi^{-}p$.I-Slope-Mass-$\text{Cos}\theta^{G.J.}$ Correlation". We calculate here the Partial Wave Amplitudes and give the results obtained for the slope-mass-partial wave correlation.

Key-words: Diffractive dissociation; Deck-model; Phenomenology.

I. INTRODUCTION

This paper is a continuation of "Diffractive Dissociation in $pp \to \Delta^{++}\pi^{-}p$.I - Slope-Mass-Cos$\theta^{G.J.}$ Correlation", henceforth called [I][1]. Here we proceed to the Partial Wave analysis for the $(\Delta^{++}\pi^{-})$ system using the helicity amplitudes and the formalism of [I]. We avoid reproducing the results of [I]. The missing formulae, expressions and notations can be found in this previous paper.

The (TCDM) gives a set of properties of the inelastic diffractive reactions. One of them is the existence of "zeros" as a consequence of the interference among its components. In some cases[2] it was possible to derive an equation for the "zeros", which determines exactly the kinematical region where they are located. But in the present case ($pp \to \Delta^{++}\pi^{-}p$) the complications arising from the spin don't allow a simple formula in which we can visualize analytically the "zeros". These "zeros" of the amplitude correspond to dips in the cross-sections that are experimentally observed in the kinematical region of energy below the threshold of the resonances.

A consequence of these "zeros" in the t_2 - distributions for the particular intervals of mass the $(\Delta^{++}\pi^{-})$ system and of $\cos\theta^{G.J.}$, is a slope(b)-mass($M_{\Delta^{++}\pi^{-}}$) - partial wave correlation.

The enhancement of the net slope of a particular wave, which decreases with increasing energies (s_1) of the dissociated particles, is an evidence of this correlation.

We examine the interferences among the components of (TCDM) and the partial waves in which they are stronger. As we could not obtain an equation to determine the position of the "zeros" we made in [I] a numerical analysis of the slope-mass-$\cos\theta^{G.J.}$ correlation. In the present paper we also calculate numerically the partial wave distributions and search for slope-mass-partial wave correlation.

In section II we give the partial wave analysis (PWA) of the helicity amplitudes obtained in [I] using the results shown in Appendix A.

We can search for interference mechanisms directly in the t_2-distributions for particular windows of the parameters. The net slope of each partial wave is a good way of checking wether the interference mechanism given by (TCDM) works or not.

In section III the results of the calculations are discussed and some conclusions are presented. In Appendix A we present the theoretical expansion for the partial waves of the two-particle final state reaction. This is done in the rest frame of (1+2) particles $(\vec{p}_1 + \vec{p}_2 = 0)$.

II. PARTIAL WAVES FOR $pp \to \Delta^{++}\pi^- p$ REACTION

We formulate in this section the (PWA) for our reaction, using the helicity amplitudes obtained in paper [I]. For our purposes, these amplitudes (defined in section IV of [I] formulae (55), (56), (57), (58)) are rewritten suitable here.

$$A_{(\pm 3/2,\pm 1/2)}(\theta,\phi) = e^{\mp i\phi}\{A^{(1)}_{(\pm 3/2,\pm 1/2)} + A^{(2)}_{(\pm 3/2,\pm 1/2)}\cos\phi + A^{(3)}_{(\pm 3/2,\pm 1/2)}\sin\phi\} \tag{1}$$

$$A_{(\mp 3/2,\pm 1/2)}(\theta,\phi) = e^{\pm 2i\phi}\{A^{(1)}_{(\mp 3/2,\pm 1/2)} + A^{(2)}_{(\mp 3/2,\pm 1/2)}\cos\phi + A^{(3)}_{(\mp 3/2,\pm 1/2)}\sin\phi\} \tag{2}$$

$$A_{(\pm 1/2,\pm 1/2)}(\theta,\phi) = \{A^{(1)}_{(\pm 1/2,\pm 1/2)} + A^{(2)}_{(\pm 1/2,\pm 1/2)}\cos\phi + A^{(3)}_{(\pm 1/2,\pm 1/2)}\sin\phi\} \tag{3}$$

$$A_{(\mp 1/2,\pm 1/2)}(\theta,\phi) = e^{\pm i\phi}\{A^{(1)}_{(\mp 1/2,\pm 1/2)} + A^{(2)}_{(\mp 1/2,\pm 1/2)}\cos\phi + A^{(3)}_{(\mp 1/2,\pm 1/2)}\sin\phi\} \tag{4}$$

where

$$\begin{cases} A^{(1)}_{(\pm 3/2,\pm 1/2)} = \pm \dfrac{i}{\sqrt{2}}\{Im_1 + a_2 Im_2 + a_3 Im_3\} & \text{(1a)} \\ A^{(1)}_{(\mp 3/2,\pm 1/2)} = \dfrac{i}{\sqrt{2}}\{Im_5 + a_2 Im_6 + a_3 Im_7\} & \text{(2a)} \\ A^{(1)}_{(\pm 1/2,\pm 1/2)} = i\,\{Im_9 + a_2 Im_{10} + a_3 Im_{11}\} & \text{(3a)} \\ A^{(1)}_{(\mp 1/2,\pm 1/2)} = \mp i\,\{Im_{13} + a_2 Im_{14} + a_3 Im_{15}\} & \text{(4a)} \end{cases}$$

$$\begin{cases} A^{(2)}_{(\pm 3/2,\pm 1/2)} = \pm \dfrac{i}{\sqrt{2}}(b_2 Im_2 + b_3 Im_3 + Im_4) & \text{(1b)} \\ A^{(2)}_{(\mp 3/2,\pm 1/2)} = \dfrac{i}{\sqrt{2}}(b_2 Im_6 + b_3 Im_7 + Im_8) & \text{(2b)} \\ A^{(2)}_{(\pm 1/2,\pm 1/2)} = i\,(b_2 Im_{10} + b_3 Im_{11} + Im_{12}) & \text{(3b)} \\ A^{(2)}_{(\mp 1/2,\pm 1/2)} = \mp i\,(b_2 Im_{14} + b_3 Im_{15} + Im_{16}) & \text{(4b)} \end{cases}$$

$$\begin{cases} A^{(3)}_{(\pm 3/2,\pm 1/2)} = -\frac{1}{\sqrt{2}}\, Re'_{(\pm 3/2,\pm 1/2)} & (1c) \\ A^{(3)}_{(\mp 3/2,\pm 1/2)} = \pm\frac{1}{\sqrt{2}}\, Re'_{(\mp 3/2,\pm 1/2)} & (2c) \\ A^{(3)}_{(\pm 1/2,\pm 1/2)} = \pm\, Re'_{(\pm 1/2,\pm 1/2)} & (3c) \\ A^{(3)}_{(\mp 1/2,\pm 1/2)} = -\, Re'_{(\mp 1/2,\pm 1/2)} & (4c) \end{cases}$$

$$Re'_{(\pm 3/2,\pm 1/2)} = \frac{1}{\sin\phi}\, Re_{(\pm 3/2,\pm 1/2)} \tag{1c-1}$$

$$Re'_{(\mp 3/2,\pm 1/2)} = \frac{1}{\sin\phi}\, Re_{(\mp 3/2,\pm 1/2)} \tag{2c-1}$$

$$Re'_{(\pm 1/2,\pm 1/2)} = \frac{1}{\sin\phi}\, Re_{(\pm 1/2,\pm 1/2)} \tag{3c-1}$$

$$Re'_{(\mp 1/2,\pm 1/2)} = \frac{1}{\sin\phi}\, Re_{(\mp 1/2,\pm 1/2)} \tag{4c-1}$$

$$a_2 = m_2^2 + m_3^2 + \frac{1}{2s_1}[(s_1 + m_2^2 - m_1^2)(s - s_1 - m_3^2) + $$

$$+ \lambda^{1/2}(s_1, m_1^2, m_2^2)\, \lambda^{1/2}(s, s_1, m_3^2) \cos\alpha \cos\theta]$$

$$b_2 = -\frac{1}{2s_1}[\lambda^{1/2}(s_1, m_1^2, m_2^2)\, \lambda^{1/2}(s, s_1, m_3^2) \sin\alpha \sin\theta]$$

$$a_3 = s - s_1 + m_1^2 + m_2^2 + m_3^2 - a_2$$

$$b_3 = -b_2$$

Let us make some comments on these formulae. We have separated all helicity amplitudes in three parts, one of them which has no dependence on ϕ, one multiplied by $\cos\phi$ and another by $\sin\phi$. This separation for (PWA) make the integration on ϕ easier to be performed as we can see below.

For each one of the coefficients $A^{(i)}_{\lambda_1\lambda_a}$, $i=1,2,3$ we make some important remarks. In order to make the observations below easier to be understood, the components of (TCDM) present in each coefficient are shown in Table (II-1).

(i) The coefficients $A^{(1)}_{(\pm 1/2,\pm 1/2)}$, $A^{(2)}_{(\pm 1/2,\pm 1/2)}$, $A^{(1)}_{(\mp 1/2,\pm 1/2)}$ and $A^{(2)}_{(\mp 1/2,\pm 1/2)}$ contain the three components of (TCDM). Consequently, if there are interferences, they are given by these coefficients.

(ii) The coefficients $A^{(1)}_{(\pm 3/2,\pm 1/2)}$, $A^{(2)}_{(\pm 3/2,\pm 1/2)}$, $A^{(1)}_{(\mp 3/2,\pm 1/2)}$, and $A^{(2)}_{(\mp 3/2,\pm 1/2)}$ contain only T and U components. Then, we cannot expect strong interferences of (TCDM) type.

(iii) The coefficients $A^{(3)}_{(\pm 3/2,\pm 1/2)}$ and $A^{(3)}_{(\mp 3/2,\pm 1/2)}$ contain only U components, so we do not expect interferences at all.

(iv) Finally the coefficients $A^{(3)}_{(\pm 1/2,\pm 1/2)}$ and $A^{(3)}_{(\mp 1/2,\pm 1/2)}$ contain U and S components. Then the possible interferences are not expect to be strong ones.

Now we write the amplitudes given by (1), (2), (3) and (4) in a general way:

$$A_{\lambda_1\lambda_a}(\theta,\phi) = e^{-i(\lambda_1-\lambda_a)\phi}\,\tilde{A}_{\lambda_1\lambda_a}(\theta,\phi) \tag{5}$$

where

$$\tilde{A}_{\lambda_1\lambda_a}(\theta,\phi) = A^{(1)}_{\lambda_1\lambda_a}(\theta) + A^{(2)}_{\lambda_1\lambda_a}(\theta)\cos\phi + A^{(3)}_{\lambda_1\lambda_a}(\theta)\sin\phi. \tag{6}$$

The integration on ϕ which appears in (A-27) can be done:

$$\int_{-\pi}^{\pi} d\phi \; e^{-i(M-\lambda_a)\phi} \tilde{A}_{\lambda_1\lambda_a}(\theta,\phi) = 2\pi \, A^{(1)}_{\lambda_1\lambda_a}(\theta)\, \delta_{M\lambda_a} + $$

$$+ \pi[A^{(2)}_{\lambda_1\lambda_a}(\theta) + i\, A^{(3)}_{\lambda_1\lambda_a}(\theta)]\, \delta_{M,\lambda_a-1} + \pi[A^{(2)}_{\lambda_1\lambda_a}(\theta) - i\, A^{(3)}_{\lambda_1\lambda_a}(\theta)]\, \delta_{M,\lambda_a+1} \tag{7}$$

Thus we may obtain partial wave amplitudes for each value of M, as it is usually done in the experimental analysis. As we want to show that there may happen interferences in the (PWA), we chose (for simplicity) only the value $M = \lambda_a$, because this choice automatically selects the amplitudes in which they are likely to occur.

The two other possible choices, $M = \lambda_a - 1$ and $M = \lambda_a + 1$, would select coefficients of the type $A^{(2)}$, which could also have strong interferences, but added to $A^{(3)}$ coefficients, what could destroy these interferences.

From (7) and (A-27), we obtain the partial wave amplitudes for well defined total angular momentum J and its projection on the incident beam momentum $M = \lambda_a$:

$$A^{J,M=\lambda_a,\pm}_{\lambda_1\lambda_a} = \sqrt{\pi(J+1/2)} \int_{0}^{\pi} \sin\theta \, d\theta \{ d^{J}_{\lambda_a\lambda_1}(\theta)\, A^{(1)}_{\lambda_1\lambda_a}(\theta) \; \pm$$

$$\pm \; d^{J}_{\lambda_a,-\lambda_1}(\theta) A^{(1)}_{-\lambda_1,\lambda_a}(\theta) \} \tag{8}$$

which fullfil the relations,

$$A^{J,-\lambda_a,\pm}_{-\lambda_1,-\lambda_a} = A'^{J,\lambda_a,\pm}_{\lambda_1,\lambda_a} \tag{9}$$

$$A^{J,\lambda_a,\pm}_{-\lambda_1,\lambda_a} = \pm A^{J,\lambda_a,\pm}_{\lambda_1,\lambda_a} \tag{10}$$

For well defined orbital angular momentum L the amplitudes are given by (A-30),

$$A^{J,M,\pm}_{(L)\lambda_a} = \sqrt{\frac{2L+1}{2J+1}}\,[1 \pm N_{12}(-1)^{J-L-\delta_1}] \sum_{|\lambda_1|} C^{L,\delta_1,J}_{o,|\lambda_1|,|\lambda_1|}\, A^{J,M,\pm}_{|\lambda_1|,\lambda_a}$$

(where $A^{J,M,\pm}_{|\lambda_1|,\lambda_a}$ is given by (8)), satisfying the relation (A-31) for which $M = \lambda_a$ gives

$$A^{J,-\lambda_a,\pm}_{(L),-\lambda_a} = \pm\, A^{J,\lambda_a,\pm}_{(L),\lambda_a} \tag{11}$$

The parity of these amplitudes is given by $P = \pm(-1)^{J-1/2}$. As our model must be applied only to a restrict range of the effective mass of the subsystem $(\Delta^{++}\pi^-)$, below the first resonance threshold $(m_\Delta + m_\pi \le \sqrt{s_1} \le m_{N*})$, we expect that in this region only the first few partial waves contribute significatively to the subreaction $\mathbb{P} + p \to \Delta^{++}\pi^-$. For convenience we denote the amplitudes used in our calculations by

$$A(L^P_J)_{1/2} = A^{J,1/2,\pm}_{(L)\,1/2} \quad . \tag{12}$$

Now using (A-40) we have the following partial wave amplitudes, one for S-wave, three for P-wave and four amplitudes for D-wave:

$$A(S^-_{3/2})_{1/2} = A^{3/2,1/2,+}_{3/2,1/2} + A^{3/2,1/2,+}_{1/2,1/2} \tag{13}$$

$$\begin{cases} A(P^+_{1/2})_{1/2} = -\sqrt{2}\, A^{1/2,1/2,+}_{1/2,1/2} & \text{(14a)} \\ A(P^+_{3/2})_{1/2} = -\frac{1}{\sqrt{5}}(3A^{3/2,1/2,-}_{3/2,1/2} + A^{3/2,1/2,-}_{1/2,1/2}) & \text{(14b)} \\ A(P^+_{5/2})_{1/2} = \frac{1}{\sqrt{5}}(2A^{5/2,1/2,+}_{3/2,1/2} + \sqrt{6}\, A^{5/2,1/2,+}_{1/2,1/2}) & \text{(14c)} \end{cases}$$

$$\begin{cases} A(D^-_{1/2})_{1/2} = \sqrt{2}\, A^{1/2,1/2,-}_{1/2,1/2} & \text{(15a)} \\ A(D^-_{3/2})_{1/2} = A^{3/2,1/2,+}_{3/2\ 1/2} - A^{3/2,1/2,+}_{1/2,1/2} & \text{(15b)} \\ A(D^-_{5/2})_{1/2} = -\sqrt{\frac{2}{7}}(\sqrt{6}\, A^{5/2,1/2,-}_{3/2,1/2} + A^{5/2,1/2,-}_{1/2,1/2}) & \text{(15c)} \\ A(D^-_{7/2})_{1/2} = \frac{1}{\sqrt{7}}(\sqrt{5}\, A^{7/2,1/2,+}_{3/2,1/2} + 3A^{7/2,1/2,+}_{1/2,1/2}) & \text{(15d)} \end{cases}$$

Based only on the expressions above we cannot predict exactly the behaviour of these amplitudes with respect to the interferences. We know that in all amplitudes (13) to (15) the terms which give the strong interferences are present. However there are other terms not trivially added that can give rise to complicated cancelations so that we cannot have any clear conclusion. Thus we made some numerical calculations that we shall comment in the next section.

The cross-section calculated from the amplitudes (13) to (15), for each partial wave (L) with well defined $|M|=1/2$, an given by (following (A-47) from [I]):

$$\frac{d\sigma^L_{|M|=1/2}}{dt_2} = c \int_{(M_{\Delta^{++}\pi^-})^2}^{(M'_{\Delta^{++}\pi^-})^2} ds_1 \frac{\lambda^{1/2}(s_1, m_1^2, m_2^2)}{s_1} \sum_J |A(L^P_J)_{1/2}|^2 \qquad (16)$$

III. RESULTS AND CONCLUSIONS

The partial wave $d\sigma/dt_2$-distributions for the $pp \to \Delta^{++}\pi^- p$ reaction were calculated. This allows us to look for the existence of a slope-mass-partial wave correlation, as observed in other cases (e.g. KN [3]).

Our results are shown in figures (1) to (6). The set of parameters used in the calculations is the same as that of paper [I]: $\sigma^{\pi N}_{Tot} = 25$ (mb), $\sigma^{NN}_{Tot} = 50.$(mb), $\sigma^{N\Delta}_{Tot} = 80$ (mb), $B_{\pi N} = 10$ (GeV^{-2}), $B_{NN} = 9.$ (GeV^{-2}) and $B_{N\Delta} = 8.$ (GeV^{-2}). Two effective mass ($M_{\Delta^{++}\pi^-}$) intervals were considered: $1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV) and $1.40 \leq$ $\leq M_{\Delta^{++}\pi} \leq 1.45$ (GeV).

For the partial wave amplitudes $A^{JM,\pm}_{(L)\lambda_a}$ with well defined total angular momentum (J) of the subsystem ($\Delta\pi$) and its projection in the direction of the incident beam (M), orbital angular momentum (L) and normality (±), we have restricted the calculations to the values $M = \lambda_a$ (or $|M| = 1/2$).

This restriction has the advantages of simplifying the calculations and making possible to choose the partial wave amplitudes in which it is expected that the strong interferences due to TCDM occur.

The condition $|M| = 1/2$ is enough to verify a possible slope-mass-partial wave correlation. This assumption was corroborated by the results presented here. We hope that these results will be confirmed experimentally.

Figure 1 shows the three partial wave (S,P and D) distributions, obtained directly by (13), (14) and (15). We can see that there is a strong interference in the S-wave, with a dip at $t_2 = -0.3(GeV^2)$. The P - wave shows two different behaviours, or two slopes, and the D-wave shows only one slope in the considered range of t_2.

An examination of each $A(L_J^P)_{1/2}$ wave distribution allows us to understand specifically which of them have strong interferences.

Figure (3) shows the P - wave distributions, i.e., the $A(P^+_{1/2})_{1/2}$, $A(P^+_{3/2})_{1/2}$ and $A(P^+_{5/2})_{1/2}$ given by (14a), (14b) and (14c) respectively. Among the P - wave amplitudes, the $A(P^+_{3/2})_{1/2}$ is the one that has the strongest interference, with a dip at $t_2 = -0.13(GeV^2)$.

The relative normalization in figure (3), shows that the partial wave $A(P^+_{1/2})_{1/2}$ is two orders of magnitude biger than $A(P^+_{3/2})_{1/2}$ where the strongest interference occurs. For this reason the total P - wave distribution shown in Fig. 1, does not present the structures of the $A(P^+_{3/2})_{1/2}$ wave.

In Fig. 5 we have the D-wave spectrum for each J, i.e., $A(D^-_{1/2})_{1/2}$, $A(D^-_{3/2})_{1/2}$, $A(D^-_{5/2})_{1/2}$ and $A(D^-_{7/2})_{1/2}$ waves given by (15a,b,c and d) equations respectively. We remark that the $A(D^-_{1/2})_{1/2}$ and $A(D^-_{7/2})_{1/2}$ present the dips at $t_2 \simeq -0.6(GeV^2)$ and $t_2 \simeq -0.3(GeV^2)$ respectively, while no dip is seen in the two other. But as we can see in Fig. 1, the D-wave has not a special behaviour. This fact is easily understood, from Fig. 5, by the difference among the absolute values of each particular wave.

Figures 2, 4 and 6 show the same behaviour that appears in Figs. 1, 3 and 5 respectively, but for a mass range farther from the threshold, $1.40 \leq M_{\Delta^{++}\pi^{-}} \leq 1.45$ (GeV).

Summarizing, as we see in Figs. 1 and 2, the P and D waves do not present dips because all the contributions from the different values of J are added. These structures appear in the P-wave only for $J = 3/2$, and in the D-wave for $J=1/2$ and $J=7/2$, but as can be seen in Figs. 3 and 4 and in Figs. 5 and 6, the dips are covered by the other values of J.

We also calculated the net slopes (b) for each wave. Table (III-1) shows the values obtained for the interval $0. \leq$ $\leq t_2 \leq 0.02$ (GeV^2) for two different ranges of invariant mass $M_{\Delta^{++}\pi^{-}}$. Table (III-2) shows the slopes for each wave with L and J well defined, calculated in the same conditions as those of Table (III-1). In this Table we remark that all waves but $P_{J=1/2}$ -wave present normal mass-slope correlation that is, the slope decreases with increasing the invariant mass ($M_{\Delta^{++}\pi^{-}}$). This abnormal behaviour of the $P_{J=3/2}$ - wave is because the zero occurs for smaller $|t_2|$ when s_1 increases.

Finally, this set of results shows that the general interferences predicted by (TCDM) are also maintained in this particular case, and give a new correlation among partial waves.

We completed in this paper the calculations started in [I]. We could not include here a comparison with experimental results because, as far as we know, they don't exist. But the amplitude derived here can be employed in a future analysis of the data.

APPENDIX A

PARTIAL WAVE EXPANSION (PWE) OF THE SUBSYSTEM (1+2) OF A GENERIC REACTION $a+b \to (1+2)+3$

In the Diffractive Dissociation Reaction where we have the Pomeron exchanged between b and 3, the helicity amplitudes decouple in the helicities of the particles b and 3 and the helicities of the dissociation vertex $a \to 1+2$ (Fig.(A1)). This fact, due to Pomeron factorization, enables us to write helicity amplitudes which do not depend on the helicities of the particles b and 3. These amplitudes are defined by,

$$A_{\lambda_1 \lambda_2 \lambda_a}(s,s_1,t_2,\theta,\phi) = \langle p\,\theta\,\phi,\lambda_1\lambda_2;\vec{p}_3|A|\vec{p}_a,\lambda_a,\ \vec{p}_b\rangle \qquad (A1)$$

where $p = |\vec{p}_1| = |\vec{p}_2|$, $\vec{p}_a$, $\vec{p}_b$, $\vec{p}_3$, θ and ϕ are defined in the Gottfried-Jackson system (Appendix A of [1]).

Our purpose in this Appendix is to develop the subsystem (1+2) in partial waves, thus the states of interest for our calculations do not suffer any influence from ($b\mathbb{P}3$) vertex. The (PWE) of the subsystem (1+2) can be made through the following steps,

i) first we define the state that has the minimal set of the quantum numbers of the subsystem (1+2),

$$|p\,\theta\,\phi,\lambda_1\lambda_2\rangle = \sum_{J,M} \sqrt{\frac{2J+1}{4\pi}}\,\mathcal{D}_{M\lambda}^{J}(\phi,\theta,-\phi)\ |p\,J\,M,\lambda_1\lambda_2\rangle \qquad (A-2)$$

where $\lambda = \lambda_1 - \lambda_2$ is the balance of helicities of the final particles 1 and 2 and $\mathcal{D}_{M\lambda}^{J}(\phi,\theta,-\phi)$ is the rotation matrix. The reverse formula is

$$|p\,J\,M,\lambda_1\lambda_2> = \sqrt{\frac{2J+1}{4\pi}} \int d\Omega\, \mathcal{D}_{M\lambda}^{J*}(\phi,\theta,-\phi)\; |p\,\theta\,\phi,\lambda_1\lambda_2> \qquad \text{(A-3)}$$

With these expression we write now the amplitudes of total angular momentum (J,M) and helicities,

$$A^{JM}_{\lambda_1\lambda_2,\lambda_a}(s,s_1,t_2) = <p\,J\,M,\lambda_1\lambda_2;\vec{p}_3|A|\vec{p}_a,\lambda_a;\vec{p}_b> \qquad \text{(A-4)}$$

ii) The (PWE) of these helicity amplitudes are written as

$$A_{\lambda_1\lambda_2,\lambda_a}(s,s_1,t_2,\theta,\phi) = \sum_{JM} \sqrt{\frac{2J+1}{4\pi}}\, \mathcal{D}_{M\lambda}^{J*}(\phi,\theta,-\phi)\, A^{JM}_{\lambda_1\lambda_2,\lambda_a}(s,s_1,t_2) \qquad \text{(A-5)}$$

and their reverse expression is given by

$$A^{JM}_{\lambda_1\lambda_2,\lambda_a}(s,s_1,t_2) = \sqrt{\frac{2J+1}{4\pi}} \int d\Omega\, \mathcal{D}_{M\lambda}^{J*}(\phi,\theta,-\phi)\; A_{\lambda_1\lambda_2,\lambda_a}(s,s_1,t_2,\theta,\phi). \qquad \text{(A-6)}$$

To define amplitudes with Parity (P) and normality $(\pm)$ well defined, we introduce the corresponding states

$$|p\,J\,M,\lambda_1\lambda_2>_{(\pm)} = \frac{1}{\sqrt{2}}\,\{|p\,J\,M,\lambda_1\lambda_2> \pm N_{12}|p\,J\,M,-\lambda_1,-\lambda_2>\} \qquad \text{(A-7)}$$

where (N_{12}) refers to the normality of the (1+2) system,

$$N_{12} = \eta_1\eta_2(-1)^{s_1+s_2-v_{12}} \qquad \text{(A-8)}$$

η_1, η_2, s_1 and s_2 are the intrinsic parities and spins of the particles 1 and 2 respectively, and $\nu_{12} = 0$, for J = integer and $\nu_{12} = 1/2$ for half integer J. The parities of the states defined above are given by

$$P = \pm (-1)^{J-\nu_{12}} \qquad \text{(A-9)}$$

Thus the helicity amplitudes corresponding to these states are

$$A^{JM,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) = \frac{1}{\sqrt{2}} \{A^{JM}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) \pm N_{12}\, A^{JM}_{-\lambda_1,-\lambda_2,\lambda_a}(s,s_1,t_2)\} \qquad \text{(A-10)}$$

Using now $\mathcal{D}^J_{M\lambda}(\phi,\theta,-\phi) = e^{-i(M-\lambda)\phi}\, d^J_{M\lambda}(\theta)$ and from (A-6) and (A-10) we have

$$A^{JM,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) = \sqrt{\frac{2J+1}{8\pi}} \int d\Omega \{e^{-i(M-\lambda)\phi}\, d^J_{M\lambda}(\theta)\, A_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2,\theta,\phi) \pm$$

$$\pm N_{12}\, e^{-i(M+\lambda)\phi}\, d^J_{M\,-\lambda}(\theta)\, A_{-\lambda_1,-\lambda_2,\lambda_a}(s,s_1,t_2,\theta,\phi)\} \qquad \text{(A-11)}$$

iii) The parity invariance in strong interactions allows us to impose restrictions on the number of indepedent A-matrix elements. In order to obtain these conditions we construct the $2 \to 3$ particles helicity amplitudes in overall (CMS). According to Ref. 4, we construct firstly two particle states with well defined J, M in the (CMS). With $\vec{p}_{12} = p_{12}\hat{z}$, the operator $H(p_{12}) = R_{o,o,o}\, Zp_{12}$ represents a boost in z-direction.

The above mentioned states are

$$|p_{12};J\,M;\,\lambda_1\lambda_2> \;=\; H(p_{12})\,|p;J\,M,\,\lambda_1\lambda_2> \;. \qquad \text{(A-12)}$$

We note that M is the helicity of the subsystem (1,2) because $J_z = \dfrac{\vec{J}\cdot\vec{p}_{12}}{p_{12}}$. As the parity operator P and the Lorentz transformation operator $H(p_{12})$ commute, and

$$P|p;J\,M,\,\lambda_1\lambda_2> \;=\; \eta_1\eta_2(-1)^{J-s_1-s_2}\;|p;J\,M,\,-\lambda_1,-\lambda_2> \qquad \text{(A-13)}$$

we have for the two particle states (A-12)

$$P|p_{12};J\,M,\,\lambda_1\lambda_2> \;=\; \eta_1\eta_2(-1)^{J-s_1-s_2}\;|p_{12};J\,M,\,-\lambda_1,-\lambda_2> \qquad \text{(A-14)}$$

we can therefore construct the states

$$|p_{12};J\,M,\lambda_1\lambda_2>_{(\pm)} \;=\; \frac{1}{\sqrt{2}}\{|p_{12};J\,M,\lambda_1\lambda_2> \pm \pm\; N_{12}|p_{12};J\,M,-\lambda_1,-\lambda_2>\} \qquad \text{(A-15)}$$

A convenient frame for our immediate purposes is obtained choosing $\vec{p}_a = \vec{p}_a(\theta_o,\phi_o)$ and $\vec{p}_{12} = p_{12}\hat{z}$ in the overall (CMS), as is shown in Fig. A-2. In this particular frame we can define three-particles helicity states, according to Refs. 4 and 5,

$$|p_{12},00;J\,M(\lambda_1\lambda_2),\lambda_3>_{(\pm)} \equiv |p_{12};J\,M,\lambda_1\lambda_2>_{(\pm)} \oplus |-p_3,\lambda_3> \qquad \text{(A-16)}$$

we have $\vec{p}_{12}(\theta{=}0,\phi{=}0) = -\vec{p}_3$, thus the helicity of these states is

$\Lambda = M - \lambda_3$.

As discussed in the begining of this Appendix, the spins of particles (b) and (3) are immaterial for our purposes, what is equivalent to assume $\lambda_b = \lambda_3 = 0$, (due to the high energy conditions).

We can now write the helicity amplitudes, in the reference frame defined by Fig. A2, and whose (PWE) is

$$<p_{12},00;J\,M(\lambda_1\lambda_2)\,|A|\,p_a,\theta_0,\phi_0;\lambda_a>_{(\pm)} =$$

$$= \sum_j \left(\frac{2j+1}{4\pi}\right) \mathcal{D}^{j}_{M\lambda_a}(\phi_0,\theta_0,-\phi_0) < p_{12},J\,M(\lambda_1\lambda_2)\,|A^j|\,p_a,\lambda_a>_{(\pm)} \tag{A-17}$$

if $\vec{j}$ (total angular momentum) and j_z are conserved quantities.

As the parity is conserved, similarly to the reactions with two particles in the final states, and remmembering that M is the helicity of the subsystem (1,2) we have

$$<p_{12},J,-M(-\lambda_1,-\lambda_2)\,|A^j|\,p_a,-\lambda_a>_{(\pm)} = \eta<p_{12},J\,M(\lambda_1,\lambda_2)\,|A^j|\,p_a,\lambda_a>_{(\pm)} \tag{A-18}$$

where,

$$\eta = \eta_a \eta_{12} (-1)^{J-s_a} \tag{A-19}$$

and

$$\eta_{12} = \eta_1 \eta_2 (-1)^{s_1+s_2-J} \tag{A-20}$$

From (A-17) and (A-18) we obtain the following symmetry relation,

$$<p_{12},00,J,-M(-\lambda_1,-\lambda_2)|A|p_a,\theta_0,0,-\lambda_a>_{(\pm)} =$$

$$= \eta(-1)^{M-\lambda_a} <p_{12},00,J\,M(\lambda_1\lambda_2)|A|p_a,\theta_0,0,\lambda_a>_{(\pm)} \quad \text{(A-21)}$$

where the production plane (defined by $\vec{p}_a$, $\vec{p}_b$ and $\vec{p}_3$) was fixed as the xz-plane, i.e., $\phi_0=0$. Returning to the Gottfried-Jackson system, the above relation reads, as we intended to obtain,

$$A^{J,-M,\pm}_{-\lambda_1,-\lambda_2,-\lambda_a}(s,s_1,t_2) = \eta(-1)^{M-\lambda_a} A^{J\,M,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) \quad \text{(A-22)}$$

and also using (A-11) and (A-22),

$$A_{-\lambda_1,-\lambda_2,-\lambda_a}(s,s_1,t_2;\theta,\phi) = \eta(-1)^{\lambda_1-\lambda_2-\lambda_a} A_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2,\theta,-\phi) \quad \text{(A-23)}$$

iv) Other relations may be given by the normality of the states (±). From (A-7) we have,

$$|p\,J\,M,-\lambda_1,-\lambda_2>_{(\pm)} = \pm N_{12}|p\,J\,M,\lambda_1,\lambda_2>_{(\pm)} \quad \text{(A-24)}$$

which implies the following relations for the amplitudes (A-10),

$$A^{J\,M,\pm}_{-\lambda_1,-\lambda_2,\lambda_a}(s,s_1,t_2) = \pm N_{12} A^{J\,M,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2). \quad \text{(A-25)}$$

v) From the relations obtained above we have the helicity am plitudes for (DDR), consequently valid for our model, in the (G.J.S.). Due to Jacob-Wick conventions used here, it has an explicit phase factor:

$$A_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2,\theta,\phi) = e^{-i(\lambda-\lambda_a)\phi}\,\tilde{A}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2,\theta,\phi) \tag{A-26}$$

where $\lambda = \lambda_1 - \lambda_2$. With this property the amplitudes (A-11) can be written as,

$$A^{JM,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) = \sqrt{\frac{2J+1}{8\pi}}\int d\Omega\; e^{-i(M-\lambda_a)\phi}\{d^J_{M\lambda}(\theta)\,\tilde{A}_{\lambda_1\lambda_2\lambda_a}(\theta,\phi) \pm$$

$$\pm\, N_{12}d^J_{M,-\lambda}(\theta)\,\tilde{A}_{-\lambda_1,-\lambda_2,\lambda_a}(\theta)\} \tag{A-27}$$

vi) <u>Helicity amplitudes for well defined J, L, N and P of the (1+2) system</u>.

From the states

$$|J\,M;L,s\rangle_{(\pm)} = \sqrt{\frac{2L+1}{2J+1}}\sum_{\lambda_1\lambda_2} C^{LsJ}_{o\lambda\lambda}\, C^{s_1 s_2 s}_{\lambda_1 -\lambda_2\lambda}\,|J\,M;\lambda_1\lambda_2\rangle_{(\pm)} \tag{A-28}$$

we define the corresponding amplitudes

$$A^{JM,\pm}_{(Ls)\lambda_a}(s,s_1,t_2) = \sqrt{\frac{2L+1}{2J+1}}\sum_{\lambda_1\lambda_2} C^{LsJ}_{o\lambda\lambda}\, C^{s_1 s_2 s}_{\lambda_1,-\lambda_2\lambda}\, A^{JM,\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2). \tag{A-29}$$

Henceforth we consider only the case $s_2 = 0$. In such a case, $s = s_1$, $\lambda_2 = 0$ and $\lambda = \lambda_1$. Using $C^{s_1 0 s_1}_{\lambda_1 0 \lambda_1} = 0$, and the relation

$C^{L\,s_1\,J}_{0\,-\lambda_1,-\lambda_1} = (-1)^{J-L-s_1}\, C^{L\,s_1\,J}_{o\lambda_1\lambda_1}$, and the equation (A-25) we obtain from (A-29),

$$A^{J\,M,\pm}_{(L)\lambda_a} = \sqrt{\frac{2L+1}{2J+1}}\,[1 \pm N_{12}(-1)^{J-L-s_1}] \sum_{|\lambda_1|} C^{L\,s_1\,J}_{o\,|\lambda_1||\lambda_1|}\, A^{J\,M,\pm}_{|\lambda_1|,\lambda_a} \qquad \text{(A-30)}$$

The expressions (A-22) to (A-30) give the relation,

$$A^{J,-M,\pm}_{(L),-\lambda_a} = \pm\, N_{12}\eta(-1)^{M-\lambda_a}\, A^{J\,M,\pm}_{(L)\lambda_a} \qquad \text{(A-31)}$$

vii) Partial wave cross-sections

The relations (A-11) and (A-29) may inverted to give

$$A_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2;\theta,\phi) = \sum_{JM}\sqrt{\frac{2J+1}{8\pi}}\,\mathcal{D}^{J*}_{M\lambda}(\phi,\theta,-\phi)\{A^{JM+}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2)+$$

$$+\,A^{JM-}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2)\} \qquad \text{(A-32)}$$

and

$$A^{JM\pm}_{\lambda_1\lambda_2\lambda_a}(s,s_1,t_2) = \sum_{L,s}\sqrt{\frac{2L+1}{2J+1}}\, C^{L\,s\,J}_{o\lambda\lambda}\, C^{s_1 s_2 s}_{\lambda_1-\lambda_2\lambda}\, A^{JM\pm}_{(Ls)\lambda_a}(s,s_1,t_2) \qquad \text{(A-33)}$$

From ortogonality relation for rotation matrices,

$$\int d\Omega\,\mathcal{D}^{J'*}_{M'\lambda}(\phi,\theta,-\phi)\,\mathcal{D}^{J}_{M\lambda}(\phi,\theta,-\phi) = \frac{4\pi}{2J+1}\,\delta_{JJ'}\delta_{MM'} \qquad \text{(A-34)}$$

we obtain

$$\int d\Omega \left| A_{\lambda_1\lambda_2\lambda_a}(\theta,\phi)\right|^2 = \frac{1}{2}\sum_{JM}\left| A^{JM+}_{\lambda_1\lambda_2\lambda_a} + A^{JM-}_{\lambda_1\lambda_2\lambda_a}\right|^2 \qquad \text{(A-35)}$$

Using now the completicity relation for Clebsh-Gordan coefficients

$$\sqrt{\frac{2L'+1}{2J+1}}\sqrt{\frac{2L+1}{2J+1}}\sum_{\lambda_1\lambda_2} C^{L's'J}_{o\lambda\lambda}\, C^{s_1 s_2 s'}_{\lambda_1,-\lambda_2,\lambda}\, C^{LsJ}_{o\lambda\lambda}\, C^{s_1 s_2 s}_{\lambda_1,-\lambda_2\lambda} = \delta_{LL'}\,\delta_{ss'}$$

(A-36)

we have,

$$\sum_{\lambda_1\lambda_2}\left| A^{JM+}_{\lambda_1\lambda_2\lambda_a} + A^{JM-}_{\lambda_1\lambda_2\lambda_a}\right|^2 = \sum_{L,s}\left| A^{JM+}_{(Ls)\lambda_a} + A^{JM-}_{(Ls)\lambda_a}\right|^2 . \qquad \text{(A-37)}$$

From (A-35) and (A-37) we obtain,

$$\sum_{\lambda_1\lambda_2}\int d\Omega \left| A_{\lambda_1\lambda_2\lambda_a}(\theta,\phi)\right|^2 = \frac{1}{2}\sum_{JM}\sum_{Ls}\left| A^{JM+}_{(Ls)\lambda_a} + A^{JM-}_{(Ls)\lambda_a}\right|^2 .$$

The corresponding cross-sections are,

$$\frac{d\sigma}{dt_2} = c\int ds_1 \frac{\lambda^{1/2}(s_1 m_1^2, m_2^2)}{s_1} \frac{1}{(2s_a+1)}\sum_{\lambda_a}\frac{1}{2}\sum_{JM}\sum_{Ls}\left| A^{JM+}_{(Ls)\lambda_a} + A^{JM-}_{(Ls)\lambda_a}\right|^2 .$$

(A-38)

In our particular case, $pp \to (\Delta^{++}\pi^-)p$ reaction where we have, $s_1 = 3/2$, $s_2 = 0$, $s_a = 1/2$, $\eta_1 = +1$, $\eta_2 = -1$, $\eta_a = +1$, $\nu_{12} = 1/2$ and consequently $N_{12} = +1$ and $\eta = +1$.

The equation (A-30) gives,

$$A^{JM-}_{(L=J-3/2)\lambda_a} = A^{JM+}_{(L=J-1/2)\lambda_a} = A^{JM-}_{(L=J+1/2)\lambda_a} = A^{JM+}_{(L=J+3/2)\lambda_a} = 0 \qquad \text{(A-39)}$$

and,

$$A^{JM+}_{(L=J-3/2)\lambda_a} = \frac{1}{2}\left\{\sqrt{\frac{2J+3}{J}}\; A^{JM+}_{3/2\ \lambda_a} + \sqrt{\frac{3(2J-1)}{J}}\; A^{JM+}_{1/2\ \lambda_a}\right\} \tag{A-40a}$$

$$A^{JM-}_{(L=J-1/2)\lambda_a} = -\frac{1}{2}\left\{\sqrt{\frac{3(2J+3)}{J+1}}\; A^{JM-}_{3/2\ \lambda_a} + \sqrt{\frac{2J-1}{J+1}}\; A^{JM-}_{1/2\ \lambda_a}\right\} \tag{A-40b}$$

$$A^{JM+}_{(L=J+1/2)\lambda_a} = \frac{1}{2}\left\{\sqrt{\frac{3(2J-1)}{J}}\; A^{JM+}_{3/2\ \lambda_a} - \sqrt{\frac{2J+3}{J}}\; A^{JM+}_{1/2\ \lambda_a}\right\} \tag{A-40c}$$

$$A^{JM-}_{(L=J+3/2)\lambda_a} = \frac{1}{2}\left\{\sqrt{\frac{2J-1}{J+1}}\; A^{JM-}_{3/2\ \lambda_a} + \sqrt{\frac{3(2J+3)}{J+1}}\; A^{JM-}_{1/2\ \lambda_a}\right\} \tag{A-40d}$$

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FIGURE CAPTIONS

Fig. 1 - $d\sigma/dt_2$ distributions for S(——), P(----) and D(-.-.-) waves in the effective mass interval $1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV).

Fig. 2 - $d\sigma/dt_2$ distributions for S(——), P(----), and D(-.-.-) waves in the effective mass interval $1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV).

Fig. 3 - $d\sigma/dt_2$ distributions for $P_{(J=1/2)}$ (——), $P_{(J=3/2)}$(----) and $P_{(J=5/2)}$ (-.-.-) waves in the same effective mass interval of the Fig. 1.

Fig. 4 - $d\sigma/dt_2$ distributions for $P_{(J=1/2)}$ (——), $P_{(J=3/2)}$(----) and $P_{(J=5/2)}$ (-.-.-) waves in the same effective mass interval of the Fig. 2.

Fig. 5 - $d\sigma/dt_2$ distributions for $D_{(J=1/2)}$ (——), $D_{(J=3/2)}$(----), $D_{(J=5/2)}$ (-.-.-) and $D_{(J=7/2)}$ (-.-.-.-) waves in the same effective mass interval of the Fig. 1.

Fig. 6 - $d\sigma/dt_2$ distributions for $D_{(J=1/2)}$ (——), $D_{(J=3/2)}$(----), $D_{(J=5/2)}$ (-.-.-) and $D_{(J=7/2)}$ (-.-.-.-) waves in the same effective mass interval of the Fig. 2.

Fig.A1 - The $a+b \rightarrow (1+2)+3$ (DDR) factorized by : (i) the Pomeron exchange or ($b\mathbb{P}3$) vertex and the dissociated subprocess $a \rightarrow 1+2$. The blob in our model is given by the three components: π-exchange, Δ^{++}-exchange and p-direct pole.

Fig.A2 - (CMS) for $p_a+p_b \rightarrow p_{12}+p_3$ reaction where $|\vec{p}_a|=|\vec{p}_b|$ and $|\vec{p}_3|=|\vec{p}_{12}|=|\vec{p}_1+\vec{p}_2|$.

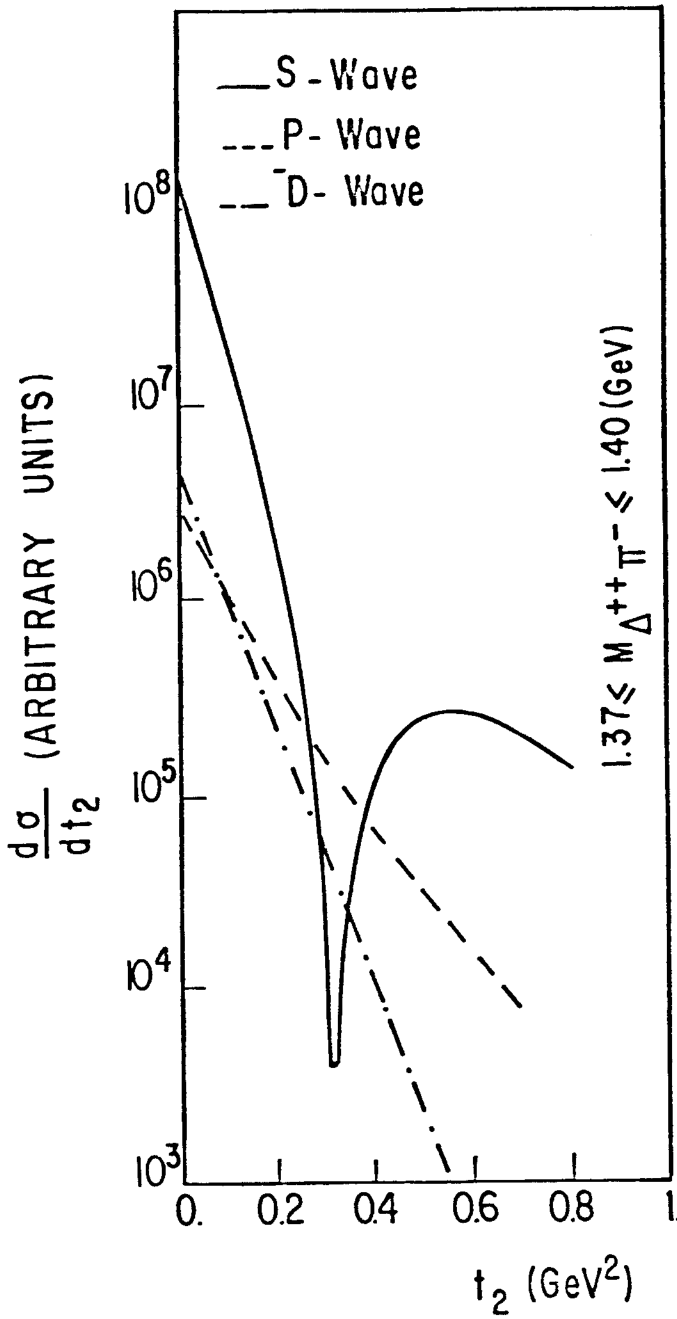
S - Wave
P - Wave
D - Wave
10^8
10^7
10^6
10^5
10^4
10^3
$\frac{d\sigma}{dt_2}$ (ARBITRARY UNITS)
$1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV)
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 1

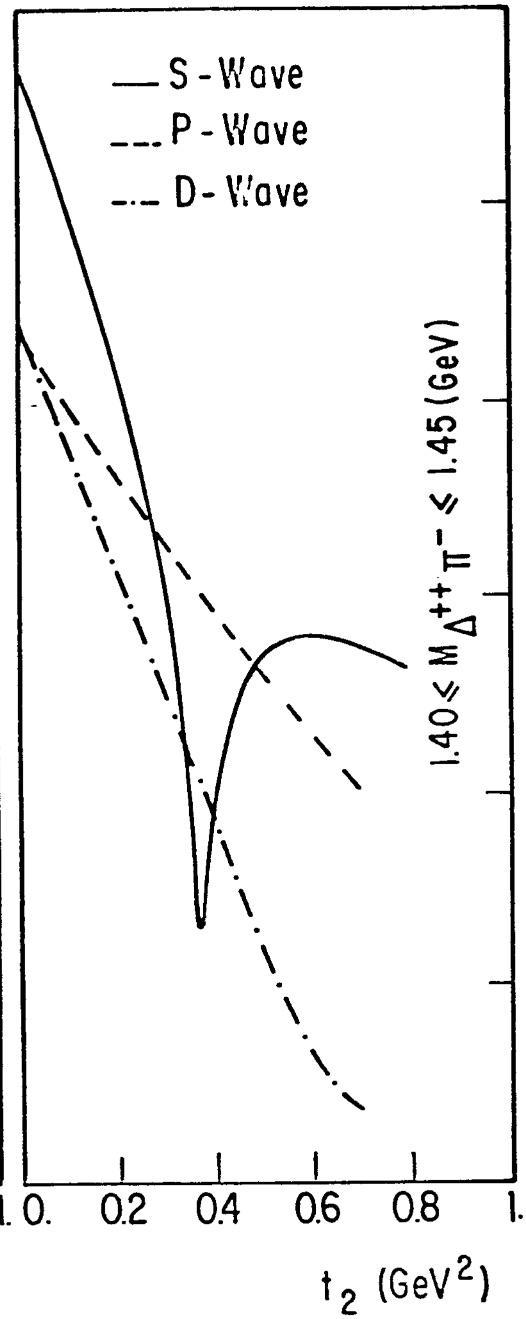
S - Wave
P - Wave
D - Wave
$1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV)
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 2

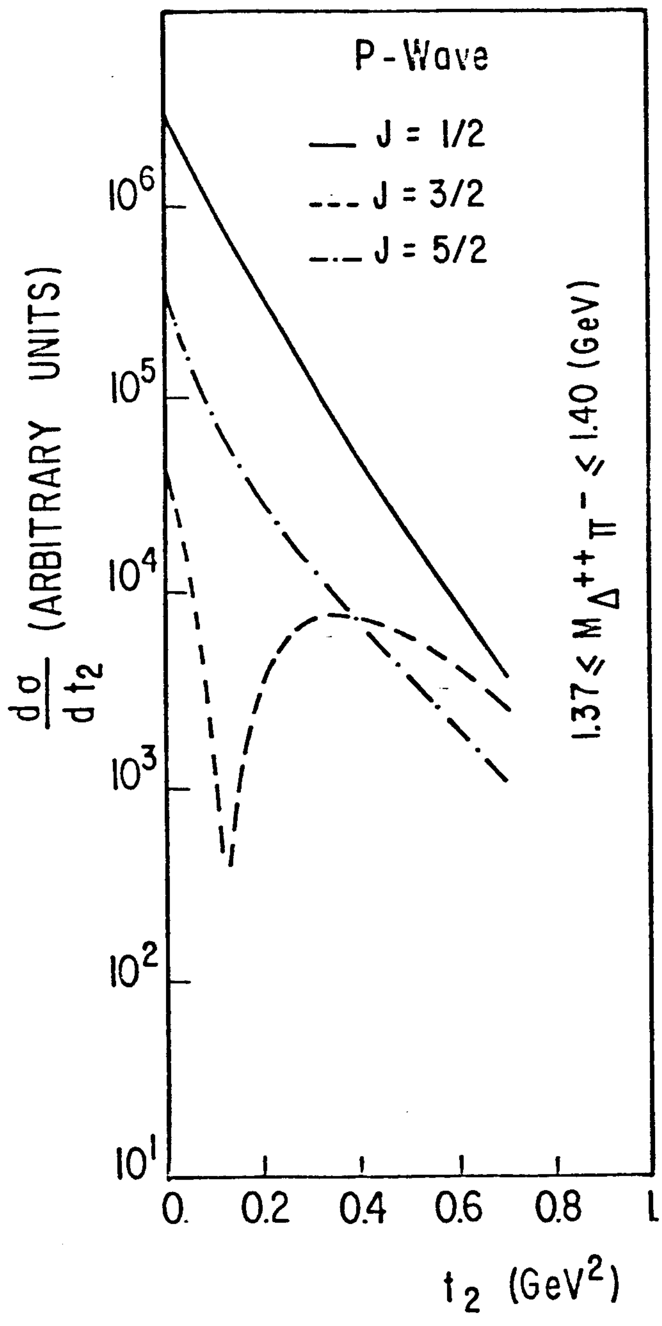
P-Wave
J = 1/2
J = 3/2
J = 5/2
10^6
10^5
10^4
10^3
10^2
10^1
$\frac{d\sigma}{dt_2}$ (ARBITRARY UNITS)
$1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV)
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 3

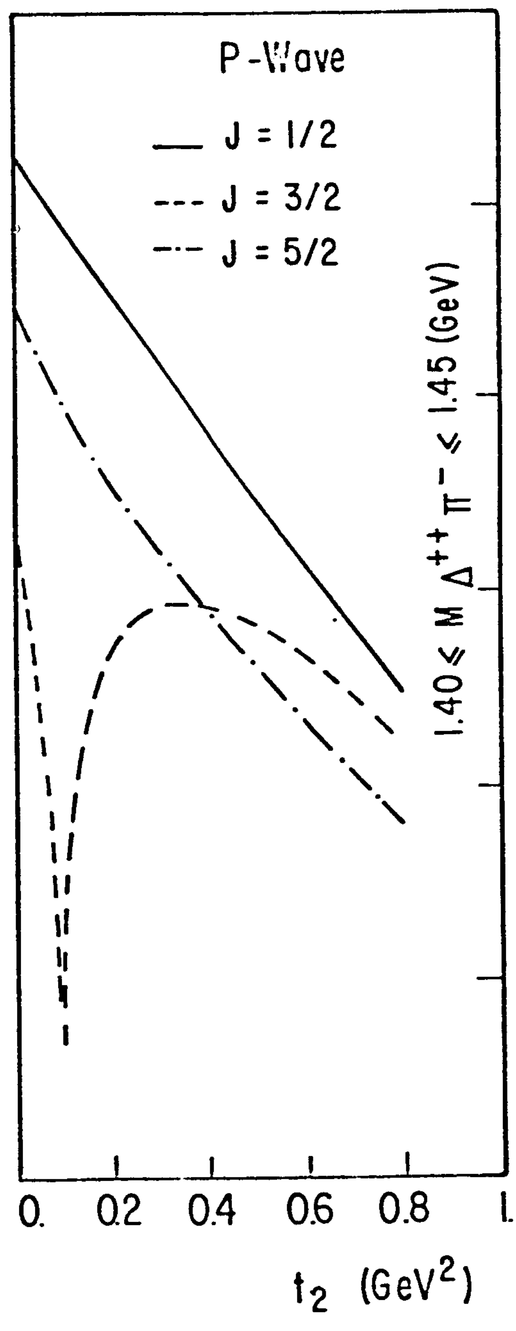
P-Wave
J = 1/2
J = 3/2
J = 5/2
$1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV)
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 4

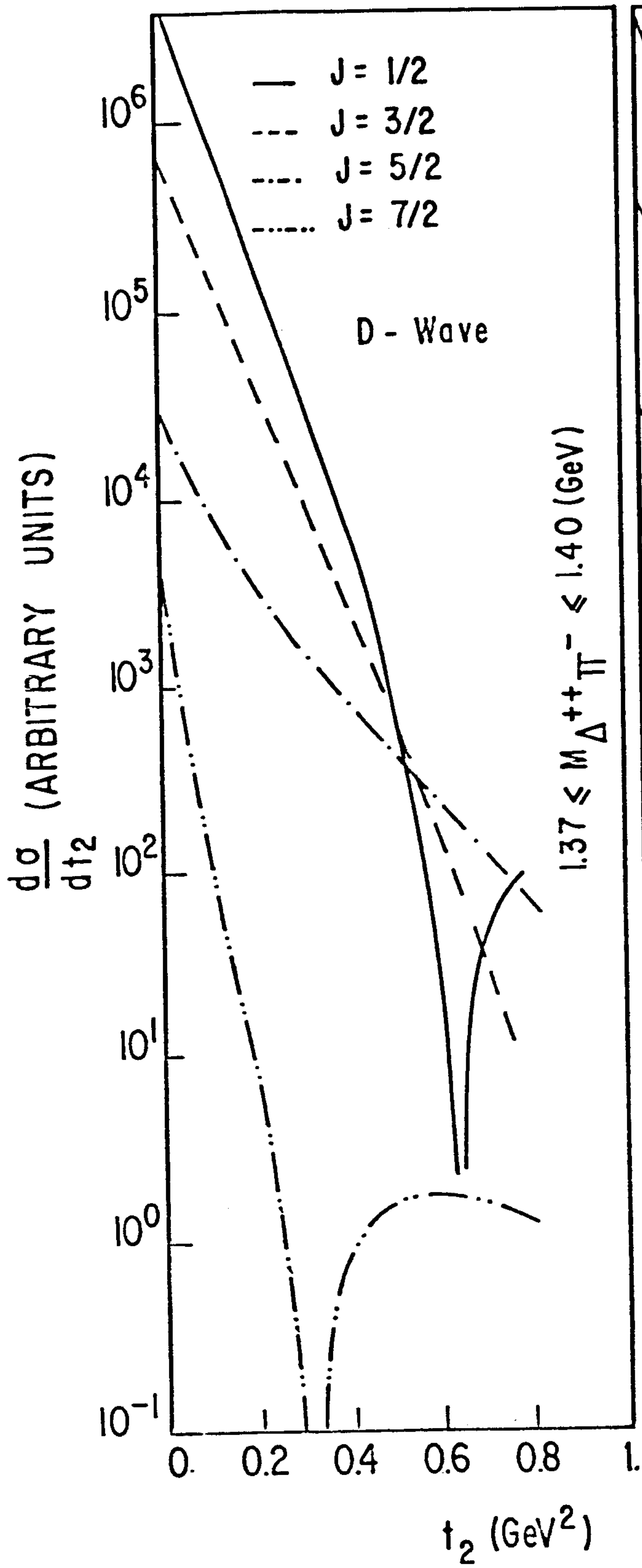
J = 1/2
J = 3/2
J = 5/2
J = 7/2
D - Wave
$1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV)
$\frac{d\sigma}{dt_2}$ (ARBITRARY UNITS)
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 5

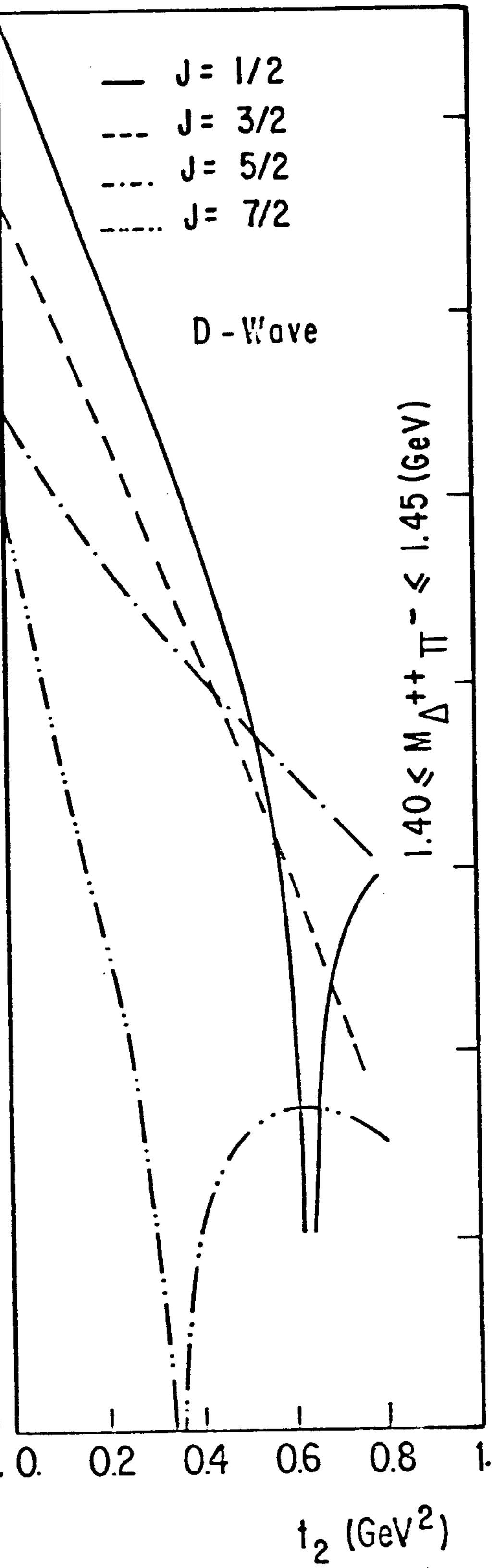
J = 1/2
J = 3/2
J = 5/2
J = 7/2
D - Wave
$1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV)
0. 0.2 0.4 0.6 0.8 1.
t_2 (GeV^2)

Fig. 6

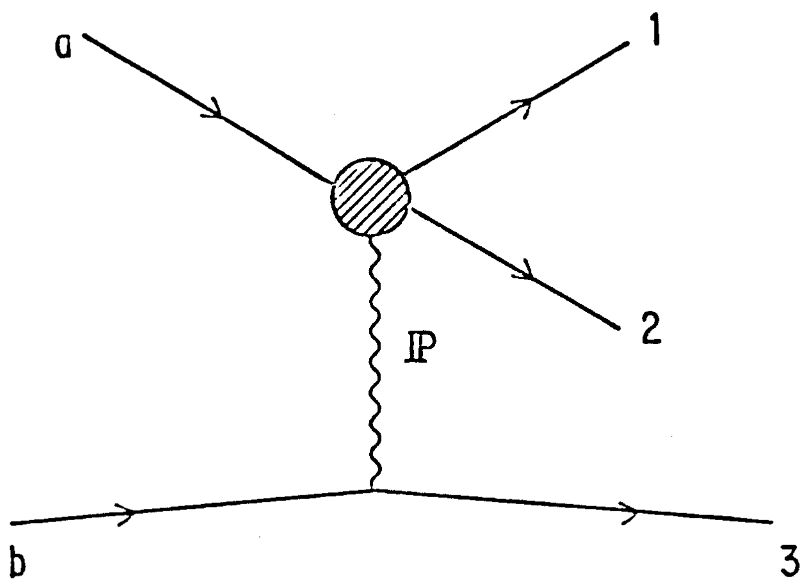

Fig. A1

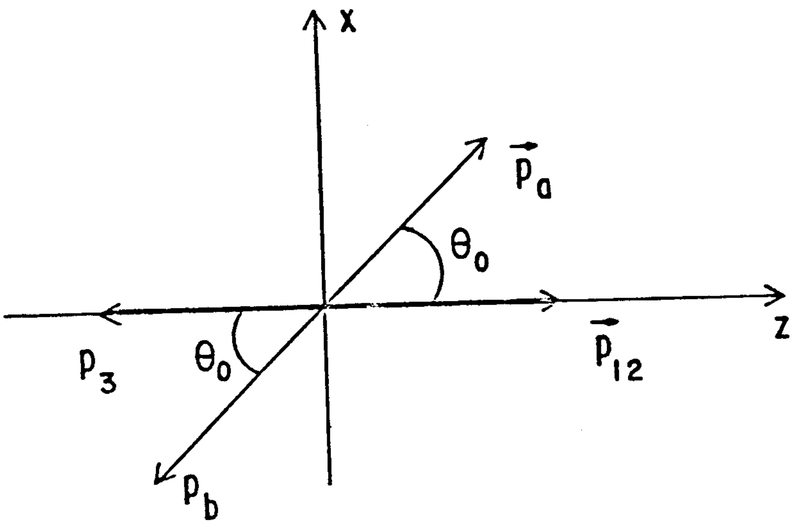

Fig. A2

Table II-1 - Components of (TCDM) present in each of the coefficients $A^{(i)}_{\lambda_1\lambda_a}$ (i=1,2,3).

$A^{(i)}_{\lambda_1\lambda_a}$		Components present in $A^{(i)}_{\lambda_1\lambda_a}$			
$\lambda_a = \pm 1/2$		U	T U	S U	S T U
$A^{(i)}$	λ_1				
$A^{(1)}$	$\pm 3/2$	no	yes	no	no
	$\mp 3/2$	no	yes	no	no
	$\pm 1/2$	no	no	no	yes
	$\mp 1/2$	no	no	no	yes
$A^{(2)}$	$\pm 3/2$	no	yes	no	no
	$\mp 3/2$	no	yes	no	no
	$\pm 1/2$	no	no	no	yes
	$\mp 1/2$	no	no	no	yes
$A^{(3)}$	$\pm 3/2$	yes	no	no	no
	$\mp 3/2$	yes	no	no	no
	$\pm 1/2$	no	no	yes	no
	$\mp 1/2$	no	no	yes	no

Table III.1 - Values of the slopes corresponding to the curves of $d\sigma/dt_2$ shown in Fig. 1 and 2.

L	$1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV)	$1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV)
S	$b = 19.6$ (GeV^{-2})	$b = 17.5$ (GeV^{-2})
P	$b = 11.2$ (GeV^{-2})	$b = 8.8$ (GeV^{-2})
D	$b = 16.1$ (GeV^{-2})	$b = 15.4$ (GeV^{-2})

Table III.2 - Values of the slopes for each wave with (L) and (J) well defined.

L	J	$1.37 \leq M_{\Delta^{++}\pi^-} \leq 1.40$ (GeV)	$1.40 \leq M_{\Delta^{++}\pi^-} \leq 1.45$ (GeV)
P	1/2	b = 9.9 (GeV^{-2})	b = 7.5 (GeV^{-2})
	3/2	b = 27.2 (GeV^{-2})	b = 32.3 (GeV^{-2})
	5/2	b = 19.9 (GeV^{-2})	b = 15.9 (GeV^{-2})
D	1/2	b = 16.5 (GeV^{-2})	b = 15.6 (GeV^{-2})
	3/2	b = 13.6 (GeV^{-2})	b = 13.3 (GeV^{-2})
	5/2	b = 16.3 (GeV^{-2})	b = 11.7 (GeV^{-2})
	7/2	b = 43.9 (GeV^{-2})	b = 35.2 (GeV^{-2})