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GLUEBALLS IN $\pi^{-}p \to \phi\phi n$

by

F. Caruso*, C.O. Escobar+, A.F.S. Santoro and
M.H.G. Souza

Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brazil

*Instituto de Física da Universidade do Estado do Rio de Janeiro
Grant from C A P E S (Brazil)

+Instituto de Física da Universidade de São Paulo, SP - Brasil

Abstract

We propose a model which is able to explain the main features of the experimental data for the reaction $\pi^- p \to \phi\phi n$, starting from the assumption that two glueball resonances with $J^{PC} = 2^{++}$ are produced in this reaction. The coupling of these glueball candidates to $\phi\phi$ is estimated, and come out to be of the same strength as ordinary hadronic couplings.

Key-words: Glueball; Strong interactions; QCD; OZI-rule.

I. INTRODUCTION

Quantum Chromodynamics has introduced in the hadronic spectroscopy the possibility of a new type of hadron, called glueballs.[1] This is an exciting subject in the development of hadronic phenomenology. There are many reviews[2] about experimental candidates, theoretical models for the production, masses, widths and quantum numbers of these objects. While we do not have any quantum field theoretical proof of the glueball existence, (nor of a hadron in QCD) some authors[3] have shown the non existence of glueballs at the classical level. These theorems[3] obviously do not forbid the existence of quantum glueballs. For us, these objects formed only by gluons must be treated as ordinary hadrons, if QCD is correct. Our main aim in this paper is to give a parametrization for a particular reaction, i.e., $\pi^- p \to \phi\phi n$, that can be used as a constraint for a microscopic model of gluon interactions producing glueballs. The choice of this reaction is related to the violation of the suppression due to the Okubo-Zweig-Iizuka (OZI)-rule[4] which forbids $(q\bar{q})$-states as possible candidates to explain the experimental results.[5] We agree with some authors[6] that the OZI-rule violation is a good condition to search for possibles glueball states.

Our phenomenological analysis is completely based on the main features of the experimental results[5] namely peripherality, and the partial wave enhancements indicating the existence of two $J^{PC} = 2^{++}$ objects, one dominantly S-wave and the other dominantly in D-wave. Our amplitude is easily constructed taking into account a production mechanism described by a single Regge parametrization, times a decay process and two hadronic $\phi\phi$ resonance propagators. These points are shown in detail in section II. In section III we present the results of our model in comparison with experimental results, and end with some conclusions.

II. THE MODEL

The peripherality of the reaction, i.e., a great number of events for small squared transferred momentum t_2, and the t_2--channel quantum numbers suggest pion exchange (see Fig. 1).

The violation of the OZI-rule hints at a glueball resonance G_i, as discussed in the introduction, with $I^G J^{PC} = 0^+ 2^{++}$ and well defined mass M_i and width Γ_i.[5] We assume that the glueball, with these quantum numbers, can be treated as a hadron, having ordinary couplings with other hadrons.

Looking at the quark diagram for the reaction $\pi^- p \to \phi\phi n$, shown in Fig. 2, we see that this reaction is OZI-forbidden, but not suppressed as shown by the experimental results.[5] The violation of the OZI-suppression can be also seen by the ratio[7];

$$\frac{\sigma(K^- p \to \phi K^+ K^- \Lambda)}{\sigma(K^- p \to \phi\phi\Lambda)} \simeq \frac{\sigma(\pi^- p \to \phi K^+ K^- n)}{\sigma(\pi^- p \to \phi\phi n)} \gtrsim 5 \ ,$$

where all the reactions, except $\pi^- p \to \phi\phi n$, are OZI-allowed, although the two ratios are the same.

Within the framework of QCD, if quarks interacts with other quarks via gluons, we can expect the existence of an amplitude of the type $q_1\bar{q}_1 \rightarrow$ gluons $\rightarrow q_2\bar{q}_2$. But the interaction among gluons in QCD can produce the new states called glueballs. The violation of the OZI-rule can then be understood qualitatively by the formation of glueballs with a strong effective coupling constant to others hadrons. This fact supports our hypothesis that glueballs couple ordinarily to other hadrons, and therefore the coupling constants $g_{G_j\phi\phi}$ must be comparable to other hadronic coupling constants.

The global amplitude representing our model is given by the expression;

$$A = \sum_{j=1}^{2} R(\pi^- p \rightarrow G_j n)\ \Phi_j(s_1)\ T(G_j \rightarrow \phi\phi)\ , \qquad (1)$$

where R represents the production amplitude, $\Phi_j(s_1) = [s_1 - M_j^2 + i M_j \Gamma_j]^{-1}$ is the resonance propagator, and T represents the decay amplitude for $G_j \rightarrow \phi\phi$.

The characteristics of the subreaction $\pi^- p \rightarrow G_j n$ discussed above permit us to treat it as a high energy $2 \rightarrow 2$ reaction, well described by a standard π-exchange Reggeised amplitude,

$$R(\pi^- p \rightarrow G_j n) = g_{\pi^+ pn}\, g_{G_j \pi^+\pi^-} \left[1+\xi \exp(-i\pi\,\alpha_\pi(t_2))\right] \left(\frac{s}{s_o}\right)^{\alpha_\pi(t_2)} \Gamma(-\alpha_\pi(t_2)) \qquad (2)$$

where $g^2_{\pi^+ pn}/4\pi = 14.5$, $\alpha_\pi(t_2) = 0.72\,(t_2 - m_\pi^2)$, $s_o = 1\,\mathrm{GeV}^2$ and $\xi = +1$. To avoid unessential complications we take into account the spin only in the decay amplitude.

In order to construct the decay amplitude $T(G_j \rightarrow \phi\phi)$ taking into account the spin-parity (J^P) of the envolved particles, we have used the helicity formalism. This amplitude is given by

$$T^M_{\lambda_1\lambda_2} = \varepsilon^{*\lambda_1}_{\mu}(\theta,\phi,p_1)\, \varepsilon^{*\lambda_2}_{\nu}(\theta,\phi,p_2)\, C^{\alpha\beta\mu\nu}\, \varepsilon^{M}_{\alpha\beta}(\vec{p}_{12}=0) \tag{3}$$

where λ_1, λ_2 are the helicities of the particles 1 and 2 $(\phi\ \phi)$, $\varepsilon^{\lambda_1(\lambda_2)}_{\mu(\nu)}$ and $\varepsilon^{M}_{\alpha\beta}$ are the spin one and spin two wave functions, respectively, defined by:[8]

$$\varepsilon^{*\pm}(\theta,\phi=0,p) = \varepsilon^{*\pm}(p_i) = \frac{1}{\sqrt{2}}(0;\mp\cos\theta, i, \pm\sin\theta) \tag{4}$$

$$\varepsilon^{0}(p_i) = \frac{1}{m_i}(p_i;\, E_i\sin\theta, 0, E_i\cos\theta) \tag{5}$$

where $i = 1,2;\ m_1 = m_2 = m_\phi$ and we have used the phase convention of reference 9. Using the fact that only $M = 0$ states were observed[5] we define the tensor $\varepsilon^{M=0}_{\alpha\beta}$, by:[8]

$$\varepsilon^{0}_{\alpha\beta}(\vec{p}_{12}=0) = \frac{1}{\sqrt{6}}\,[\varepsilon_\alpha^{+}(\vec{p}_{12}=0)\varepsilon_\beta^{-}(\vec{p}_{12}=0) + \varepsilon_\alpha^{-}(\vec{p}_{12}=0)\varepsilon_\beta^{+}(\vec{p}_{12}=0) +$$

$$+ 2\varepsilon^{0}_{\alpha}(\vec{p}_{12}=0)\ \varepsilon^{0}_{\beta}(\vec{p}_{12}=0)] \tag{6}$$

For the vertex $(2^+ \to 1^- + 1^-)$ the most general vertex function $C_{\alpha\beta\mu\nu}$ is given by the expression,[10]

$$C_{\alpha\beta\mu\nu} = g_1\, g_{\alpha\mu}\, g_{\beta\nu} + g_2(g_{\alpha\mu}\Lambda_\nu + g_{\alpha\nu}\Lambda_\mu)\Lambda_\beta + (g_3\, g_{\mu\nu} + g_4\,\Lambda_\mu\Lambda_\nu)\,\Lambda_\alpha\Lambda_\beta \tag{7}$$

Here we have used the normality of the $\phi\phi$-state, $N_{\phi\phi} = +1$, the fact that we have two identical particles, and $\Lambda_\alpha = \frac{1}{2}(p_1 - p_2)_\alpha$.

Thus the decay amplitude depends upon four constants multiplying the differents couplings of the vertex function $C_{\alpha\beta\mu\nu}$. We will see below how we can use some experimental constraints to determine the coupling constants $g_{G_T\phi\phi}$ and $g_{G_T',\phi\phi}$. Firstly we will relate the differents g_i (i=1,2,3,4), and henceforth all calculations will be made in the Gottfried-Jackson (G.J.) frame (as is shown in Fig. 3) with $\phi=0$.

Inserting (6), (7) and the Lorentz condition $\varepsilon_\mu(p)p^\mu=0$ into (3), we find,

$$\begin{aligned} T^{M=0}_{\lambda_1\lambda_2} = \frac{1}{\sqrt{6}} \{ & g_1[(\varepsilon^+.\varepsilon^*_{\lambda_1})(\varepsilon^-.\varepsilon^*_{\lambda_2}) + (\varepsilon^-.\varepsilon^*_{\lambda_1})(\varepsilon^+.\varepsilon^*_{\lambda_2}) + \\ & + 2(\varepsilon^0.\varepsilon^*_{\lambda_1})(\varepsilon^0.\varepsilon^*_{\lambda_2})] + g_2\{[(\varepsilon^+.\varepsilon^*_{\lambda_1})(\varepsilon^-.\Lambda) + (\varepsilon^-.\varepsilon^*_{\lambda_1})(\varepsilon^+.\Lambda) + \\ & + 2(\varepsilon^0.\varepsilon^*_{\lambda_1})(\varepsilon^0.\Lambda)] \frac{1}{2}(\varepsilon^*_{\lambda_2}.p_1) - [(\varepsilon^+.\varepsilon^*_{\lambda_2})(\varepsilon^-.\Lambda) + \\ & + (\varepsilon^-.\varepsilon^*_{\lambda_2})(\varepsilon^+.\Lambda) + 2(\varepsilon^0.\varepsilon^*_{\lambda_2})(\varepsilon^0.\Lambda)] \frac{1}{2}(\varepsilon^*_{\lambda_1}.p_2)\} + \\ & + [\varepsilon^+.\Lambda)(\varepsilon^-.\Lambda) + (\varepsilon^0.\Lambda)^2] [2g_3(\varepsilon^*_{\lambda_1}.\varepsilon^*_{\lambda_2}) - \frac{1}{2} g_4 (\varepsilon^*_{\lambda_1}.p_2)(\varepsilon^*_{\lambda_2}.p_1)]\} \end{aligned} \tag{8}$$

The four independent amplitudes are obtained in a straightforward calculation of the above scalar products. They are (omitting the label M=0 from now on):

$$T_{+-} = \sqrt{\frac{3}{8}}\, g_1 \sin^2\theta \tag{9}$$

$$T_{++} = \frac{1}{\sqrt{6}} (g_1 + 2g_3 |\vec{p}_1|^2)\ P_2(\cos\theta) \tag{10}$$

$$T_{+o} = \frac{\sqrt{3}}{2} \frac{E_1}{m_\phi} (-g_1 + g_2 |\vec{p}_1|^2) \sin\theta \cos\theta \tag{11}$$

$$T_{oo} = \frac{1}{\sqrt{6}} \frac{1}{m_\phi^2} [2E_1^2(-g_1 + 2g_2|\vec{p}_1|^2) + 2g_3(|\vec{p}_1|^4 + E_1^2|\vec{p}_1|^2)$$

$$-2g_4 E_1^2 |\vec{p}_1|^4] P_2(\cos\theta) \tag{12}$$

where $P_2(\cos\theta) = (3\cos^2\theta - 1)/2$.

We can now obtain the partial wave amplitudes using the well known[8] formulae

$$T_{L s}^{JM} = \sum_{\lambda_1 \lambda_2} \left(\frac{2L+1}{4\pi}\right)^{1/2} C_{\lambda_1 -\lambda_2 \lambda}^{s_1 s_2 s} C_{o\lambda\lambda}^{L s J} 2\pi \int_{-1}^{+1} d_{M\lambda}^{J}(\theta) T_{\lambda_1 \lambda_2}^{M}(\theta) d(\cos\theta) \tag{13}$$

Inserting (9)-(12) into (13) and taking into account the symmetrization of these amplitudes $[\tilde{T}_{L s}^{JM} = (1+(-1)^{L+s}) T_{L s}^{JM}]$, we find:

$$\tilde{T}_{20}^{20}(s_1) = \beta_{20}[a_{20}g_1 + b_{20}g_2 + c_{20}g_3 + d_{20}g_4] \tag{14}$$

$$\tilde{T}_{02}^{20}(s_1) = \beta_{02}[a_{02}g_1 + b_{02}g_2 + c_{02}g_3 + d_{02}g_4] \tag{15}$$

$$\tilde{T}_{22}^{20}(s_1) = \beta_{22}[a_{22}g_1 + b_{22}g_2 + c_{22}g_3 + d_{22}g_4] \tag{16}$$

$$\tilde{T}_{42}^{20}(s_1) = \beta_{42}[a_{42}g_1 + b_{42}g_2 + c_{42}g_3 + d_{42}g_4] \tag{17}$$

The $a_{L s}$, $b_{L s}$, $c_{L s}$, $d_{L s}$ and $\beta_{L s}$ are given in Table I, in terms of the invariants.

Using now the experimental results[5], i.e., only the waves $J^P L s M = 2^+020$ and 2^+220 give significant contributions to the states G_T and G_T', respectively, we can obtain constraints between the g's. We choose to relate g_2, g_3 and g_4 to g_1 setting $\tilde{T}_{20} = \tilde{T}_{22} = \tilde{T}_{42} = 0$ for the G_T state, and $\tilde{T}_{02} = \tilde{T}_{20} = \tilde{T}_{42} = 0$ for the $G_{T'}$ state.

The next step consists in replacing the obtained values for g_2, g_3 and g_4 into expression (18) to calculate g_1:

$$M\Gamma = \frac{|\vec{p}_1|}{8\pi\sqrt{s_1}} \sum_{L,s} \frac{|T^{JM}_{Ls}|^2}{4\pi} M_N^2 , \tag{18}$$

In expression (18) M_N^2 (squared nucleon mass) has been introduced so that $|T^{JM}_{Ls}|^2$ becomes dimensionless[11]. We have, for each state G_j, the expression as,

$$\Gamma_{G_T} = \frac{M_N^2}{16\pi}\sqrt{\frac{x_1}{s_1}}\left(\frac{g^2_{G_T\phi\phi}}{4\pi}\right)\beta^2_{02}[a_{02} + b_{02}\gamma_2 + c_{02}\gamma_3 + d_{02}\gamma_4]^2 \tag{19a}$$

and

$$\Gamma_{G_T'} = \frac{M_N^2}{16\pi}\sqrt{\frac{x_1}{s_1}}\left(\frac{g^2_{G_T'\phi\phi}}{4\pi}\right)\beta^2_{22}[a_{22} + b_{22}\gamma_2' + c_{22}\gamma_3' + d_{22}\gamma_4']^2 \tag{19b}$$

where, $x_1 = s_1 - 4m_\phi^2$ and,

$$\gamma_2 = \frac{4}{x_1}\left(\frac{6m_\phi^3 - \sqrt{s_1}\,(s_1 + 4m_\phi^2)}{\sqrt{s_1}(-s_1 + 4m_\phi\sqrt{s_1} - m_\phi^2)}\right) \tag{20}$$

$$\gamma_3 = \frac{2}{x_1}\left(\frac{x_1 + 4m_\phi\sqrt{s_1}}{4m_\phi\sqrt{s_1} - s_1 - m_\phi^2}\right) \tag{21}$$

$$\gamma_4 = \frac{112}{x_1^2}\left(\frac{m_\phi^2}{s_1}\right)\left(\frac{x_1 + 4m_\phi\sqrt{s_1}}{s_1 - 4m_\phi\sqrt{s_1} + m_\phi^2}\right) \tag{22}$$

$$\gamma_2' = \frac{4}{x_1}\left(\frac{9m_\phi^3 + \sqrt{s_1}\,(s_1 - 4m_\phi\sqrt{s_1} - m_\phi^2)}{\sqrt{s_1}\,(s_1 - 2m_\phi\sqrt{s_1} - m_\phi^2)}\right) \tag{23}$$

$$\gamma_3' = \frac{2}{x_1}\left(\frac{6m_\phi\sqrt{s_1} - s_1 - 6m_\phi^2}{s_1 - 2m_\phi\sqrt{s_1} - m_\phi^2}\right) \tag{24}$$

$$\gamma_4' = \frac{80}{x_1^2}\left(\frac{m_\phi^2}{s_1}\right)\left(\frac{8m_\phi^2 - s_1 - 2m_\phi\sqrt{s_1}}{s_1 - 2m_\phi\sqrt{s_1} - m_\phi^2}\right) \tag{25}$$

Introducing the branching ratio $\eta_i = \Gamma_i/\Gamma_{total}$ for each channel i considered in the decay process, and inserting into (18) and (19) the values of the parameters: $M_{G_T} = 2.18\,\text{GeV}$, $M_{G_T'} = 2.35\,\text{GeV}$; $\Gamma_{G_T'} = 0.28\,\text{GeV}$ and $\Gamma_{G_T'} = 0.32\,\text{GeV}$, we obtain

$$g_{G_T\phi\phi} = 13.7\sqrt{\eta_{G_T\to\phi\phi}} \tag{26}$$

$$g_{G_T'\phi\phi} = 2.8\sqrt{\eta_{G_{T'}\to\phi\phi}} \tag{27}$$

We do not have experimental values for $\eta_{G_T(G_{T'})\to\phi\phi}$, however, using some naive arguments to be discussed below, we obtain what we think is a reasonable bound for $\eta_{G_T(G_{T'})}$.

The number of possible channels for a decay of $J^{PC} = 2^{++}$ objects, taking into account that the glueball is a flavour singlet, is considerable. If the flavour independence is confirmed, we can expect at least the G_j-decays into other pairs of non strange vector mesons of the same $SU_F(3)$ nonet ($\rho\rho,\omega\omega$). Then neglecting the different factors coming from phase space, we get $\eta_{G_T(G_T')} \le 1/3$. Consequently, from (26) and (27)

$$g_{G_T\phi\phi} \lesssim 7.9 \tag{28}$$

$$g_{G_T'\phi\phi} \lesssim 1.6 \tag{29}$$

and we can see that these values are comparable to ordinary hadronic coupling constants.

On the other hand if by some as yet unknown dynamical reason the $\phi\phi$ channel is favoured ($\eta_{G_T(G_T')\to\phi\phi} \simeq 1$) we can understand this apparent violation of the flavour independence as indicating that glueball production in processes described by an allowed diagram is strongly suppressed (even if it has the right quantum numbers) with respect to the production of such states in processes described by forbidden diagrams which are experimentally seen not to be suppressed.

We believe that this point is very important to the study of the glueball phenomenology. A possible partial confirmation of this statement is the recent study of the reaction $\pi^- p \to \pi^+\pi^- n$,[12] indicating that the $M_{\pi^+\pi^-}$ spectrum does not show any structure in the glueball mass range.

Finally our amplitude is obtained by replacing (2), (15) and (16) in (1),

$$|A|^2 = \frac{1}{4\pi} |R(\pi^- p \to Gn)|^2 \, [\, |\tilde{T}^{20}_{02}(G_T \to \phi\phi)\Phi_{G_T}|^2 + r^2 |\tilde{T}^{20}_{22}(G_{T'} \to \phi\phi)\Phi_{G_T}|^2] \tag{30}$$

where $r \equiv g_{G_T'\pi\pi}/g_{G_T\pi\pi}$.

III. RESULTS AND CONCLUSIONS

To obtain our results we have used the expression (30) in the differential cross-sections defined by

$$\frac{d\sigma}{dM_{\phi\phi}dt_2} = 2M_{\phi\phi}[2^{10}\pi^4\lambda(s,m_\pi^2,m_p^2)]^{-1}\,\frac{\lambda^{1/2}(s_1,m_\phi^2,m_\phi^2)}{s_1}|A|^2 \qquad (31)$$

where the terms in front of $|A|^2$ come from phase space and flux factors. $\lambda(x,y,z) = x^2+y^2+z^2-2(xy+xz+yz)$.

In order to give the curves presented in figures (4) to (6), we have integrated (31) in the limits given below. As we have discussed in section II, we set $\eta_{G_T\to\phi\phi} = \eta_{G_T'\to\phi\phi}$, $r = g_{G_T'\pi\pi}/g_{G_T\pi\pi} = 0.6$ and all masses, widths as given in section II.

$d\sigma/dM_{\phi\phi}$ - TOTAL INVARIANT MASS ($M_{\phi\phi}$) DISTRIBUTION

Figure 4 shows this distribution obtained from expression (31), integrated in t_2 between the limits $-1 \leq t_2 \leq 0$. We remark that the region of the two states G_T and $G_{T'}$ is well described by the model, while for masses $M_{\phi\phi} \gtrsim 2.4$ GeV the comparison is not so good. The model with two hadronic resonances near the threshold is not able to saturate the spectrum at high masses. The little enhancement observed between 2.4 and 2.6 GeV come from spin factors in $\gamma_i'(s_1)$.

$d\sigma^L/dM_{\phi\phi}$ - PARTIAL WAVE DISTRIBUTIONS FOR $M_{\phi\phi}$

Figure 5 presents the distributions in invariant mass $M_{\phi\phi}$ for each partial wave given by the experimental results of references 5a,b, obtained from (31) with the same set of parameters used to obtain Fig. 4. We remark that the s-wave presents the same shortcomings observed in Fig. 4. Perhaps the shortcomings, in both cases, may be caused by the fact that we have neglected the s-wave for the G_T' state.

$d\sigma/dt_2$ - SQUARED MOMENTUM TRANSFER (t_2) DISTRIBUTION

We present in Fig. 6 the distribution $d\sigma/dt_2$, showing the periferal character of the studied reaction. The slope obtained by our model is $b = 15.6\ GeV^{-2}$, calculated for $0.1 \lesssim t_2 \lesssim 0.3\ GeV^2$ and $2.04 \leq M_{\phi\phi} \leq 3.0\ GeV$, while the experimental results give $b = 9.4 \pm 0.7\ GeV^{-2}$ from reference 5a,b, and $b = 12 \pm 2\ GeV^{-2}$ from reference 5c. It is clear that if we slightly vary the value of α' we can obtain slope closer to the experimental values, but it is not our aim to present a perfect fit to the experimental results.

Our curves presented in Fig. 4-6 do not have absolute normalization. If we had more accurated values for cross-sections and branching ratios we could predict the value of the coupling constants $g_{G_T(G_T')\pi\pi}$.

Let us now make some few general comments. The good agreement with experimental data shows that our aim in this paper have been attained. We think that our amplitude can be used by the experimentalists for a best determination of the involved parameters. Of course it would be extremely desirable to have other channels, such as $\omega\omega$ and $\rho\rho$ observed, in order to clarify the question about branching ratios and flavourless assumption for glueball decay. Among the candidates (see reviews in Ref. 2) for glueball states we believe, in agreement with the authors of Ref. 5a,b, that the reaction $\pi^- p \to \phi\phi n$ with $\phi\phi$-states is a good place to search for these objects because the violation of the OZI-suppression. Other experiments also shows some structures in the $M_{\phi\phi}$ spectrum in $\phi\phi$ inclusive production in π Be and pp interactions.[13]

We once again stress the fact that we have started the construction of our model, taking into account the peripheral nature of the data being in disagreement with other authors[14] who use a central mechanism for studying this process. However an interesting question is the possibility of centrally producing these objects,[15,3,i] since this could throw light on the coupling of a pomeron to a glueball, thus making possible a test of the old conjecture of a glueball-pomeron identity[16]. This brings to mind the related question of which place would a glueball occuppy in the standard Regge phenomenology. As the glueball is as good a hadron as any other quark-made hadron, we may ask where is the glueball Regge trajectory? The mechanism related to this problem, i.e., the double-pomeron exchange (as $\gamma\gamma$ interactions) permits also the important test of flavourless assumption for glueball decay. We call attention that this subject is in a certain sense related to our proposition that stablishes a glueball suppression in allowed diagrams as discussed in the end of section two.

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FIGURE CAPTIONS

Fig. 1 - Diagram representing the $\pi^- p \rightarrow \phi\phi n$ with a π-exchanged and a Glueball or a $(\phi\phi)$-resonance in the s_1-channel $s = (p_a + p_b)^2$, $s_1 = (p_1 + p_2)^2$ and $t_2 = (p_b - p_3)^2$.

Fig. 2 - Quark diagram for the reaction $\pi^- p \rightarrow \phi\phi n$.

Fig. 3 - The Gottfried - Jackson frame used in our calculation.

Fig. 4 - Total invariant mass distribution obtained from (31), as described in the text, in comparison with the data from Ref. 5a,b.

Fig. 5 - $M_{\phi\phi}$ distributions for S-wave and D-wave, respectively, in comparison with the data from Ref. 5a,b.

Fig. 6 - $d\sigma/dt_2$ distribution obtained from (31), as described in the text.

TABLE I. Value of the coefficients of the equations (14)-(17) in terms of the invariants, where $x_1 \equiv s_1 - 4m_\phi^2$.

$L\ell$	$a_{L\ell}$	$b_{L\ell}$	$c_{L\ell}$	$d_{L\ell}$	$\beta_{L\ell}$
20	$s_1 + 4m_\phi^2$	$-s_1x_1/2$	$x_1(6m_\phi^2 - s_1)$	$s_1x_1^2/16$	$\frac{2}{3m_\phi^2}\sqrt{\frac{\pi}{10}}$
02	$2(14m_\phi^2 - s_1 - 6m_\phi^2\sqrt{s_1})$	$(s_1 + 3m_\phi\sqrt{s_1})x_1$	s_1x_1	$-s_1x_1^2/8$	$\frac{1}{15m_\phi^2}\sqrt{\pi}$
22	$2(10m_\phi^2 + s_1 + 3m_\phi\sqrt{s_1})$	$-x_1(2s_1 + 3m_\phi\sqrt{s_1})/2$	$-s_1x_1$	$s_1x_1^2/8$	$\frac{2}{3m_\phi^2}\sqrt{\frac{\pi}{70}}$
42	$4m_\phi\sqrt{s_1} - x_1$	$x_1(s_1 - 2m_\phi\sqrt{s_1})/2$	$s_1x_1/2$	$s_1x_1^2/16$	$\frac{4}{5m_\phi^2}\sqrt{\frac{\pi}{14}}$

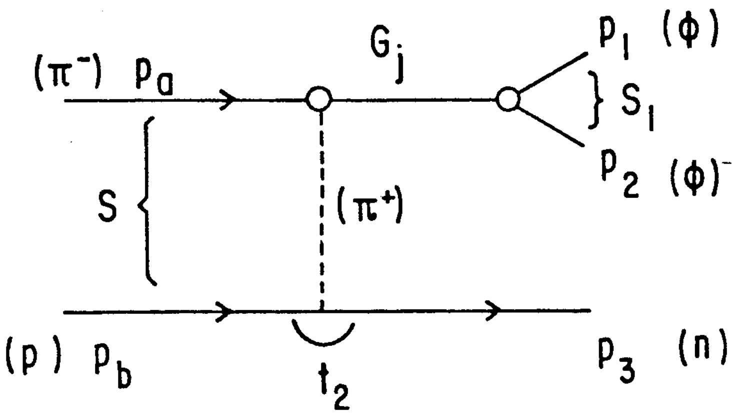

Fig. 1

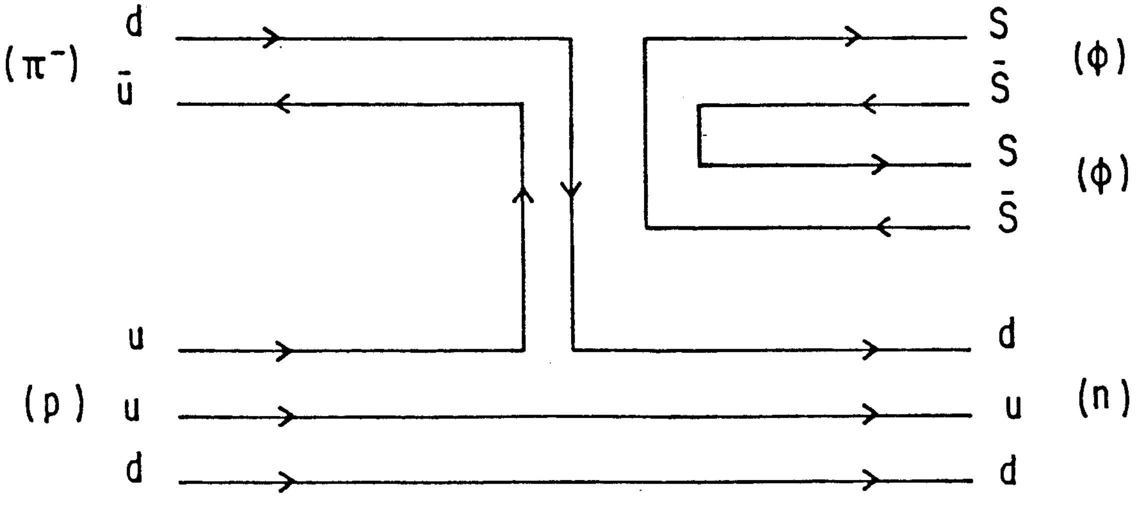

Fig. 2

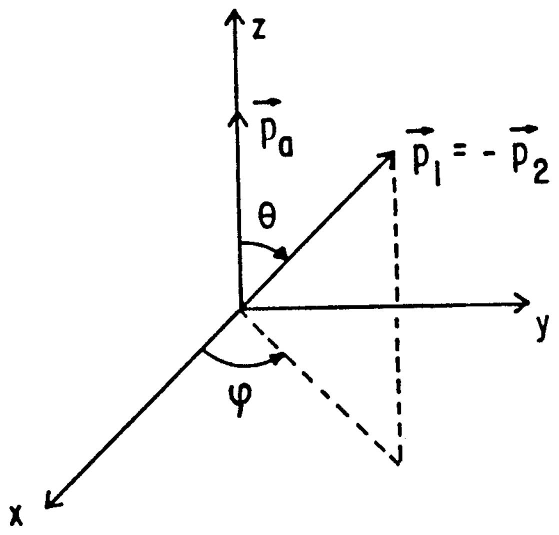

Fig. 3

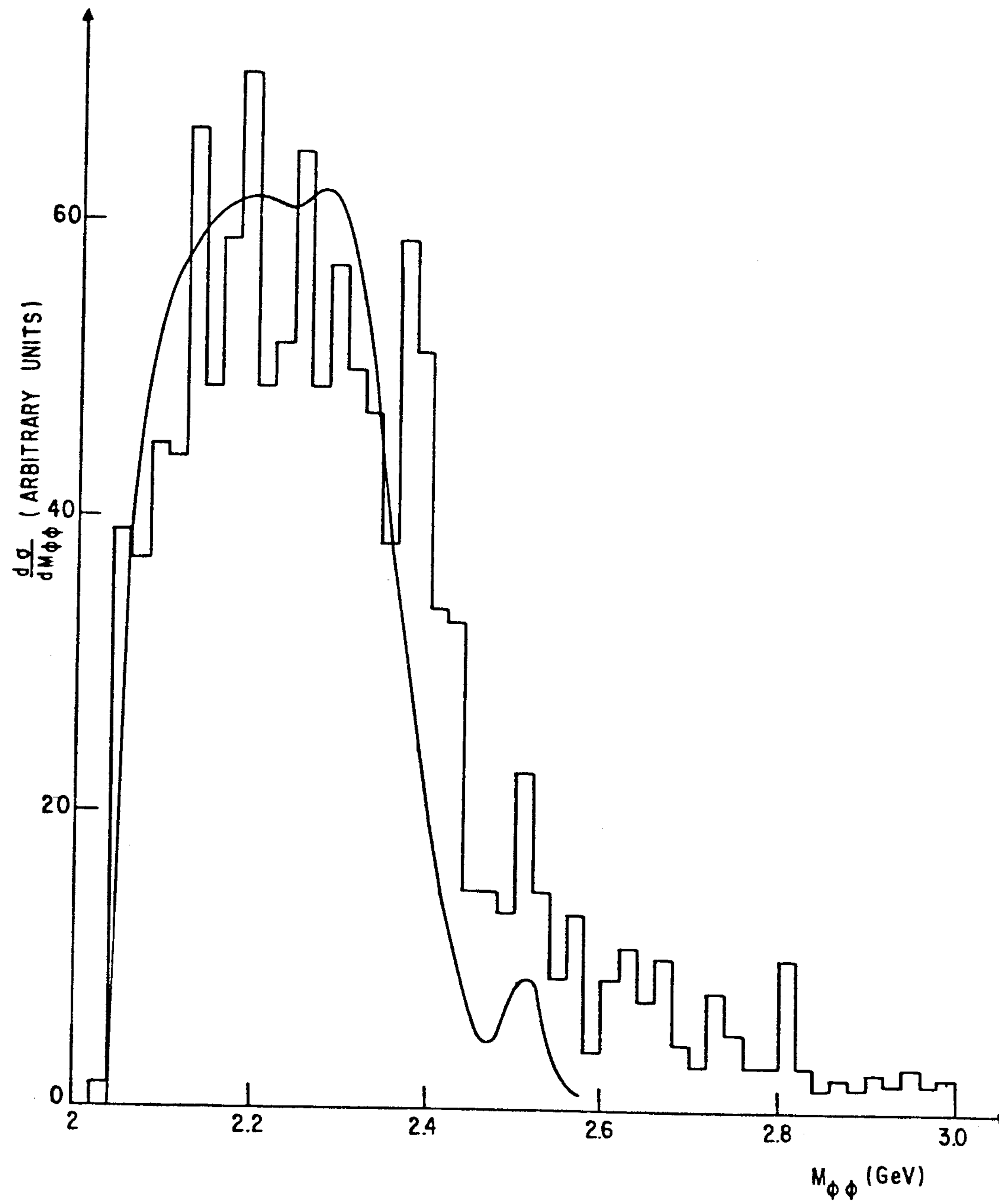
$\frac{d\sigma}{dM_{\phi\phi}}$ (ARBITRARY UNITS)
60
40
20
0
2
2.2
2.4
2.6
2.8
3.0
$M_{\phi\phi}$ (GeV)

Fig. 4

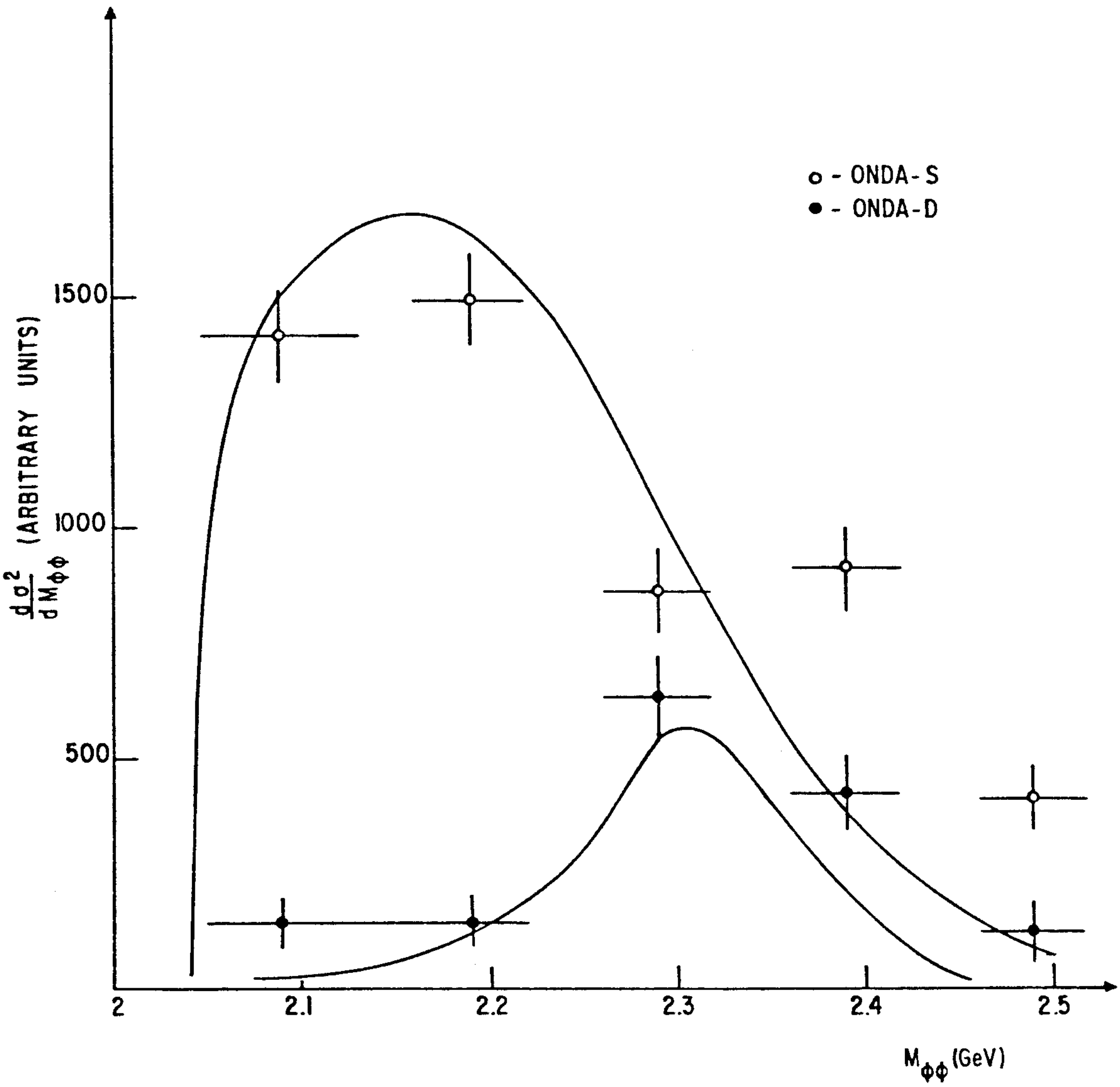
o - ONDA-S
● - ONDA-D
1500
1000
500
$\frac{d\sigma^2}{dM_{\phi\phi}}$ (ARBITRARY UNITS)
2
2.1
2.2
2.3
2.4
2.5
$M_{\phi\phi}$ (GeV)

Fig. 5

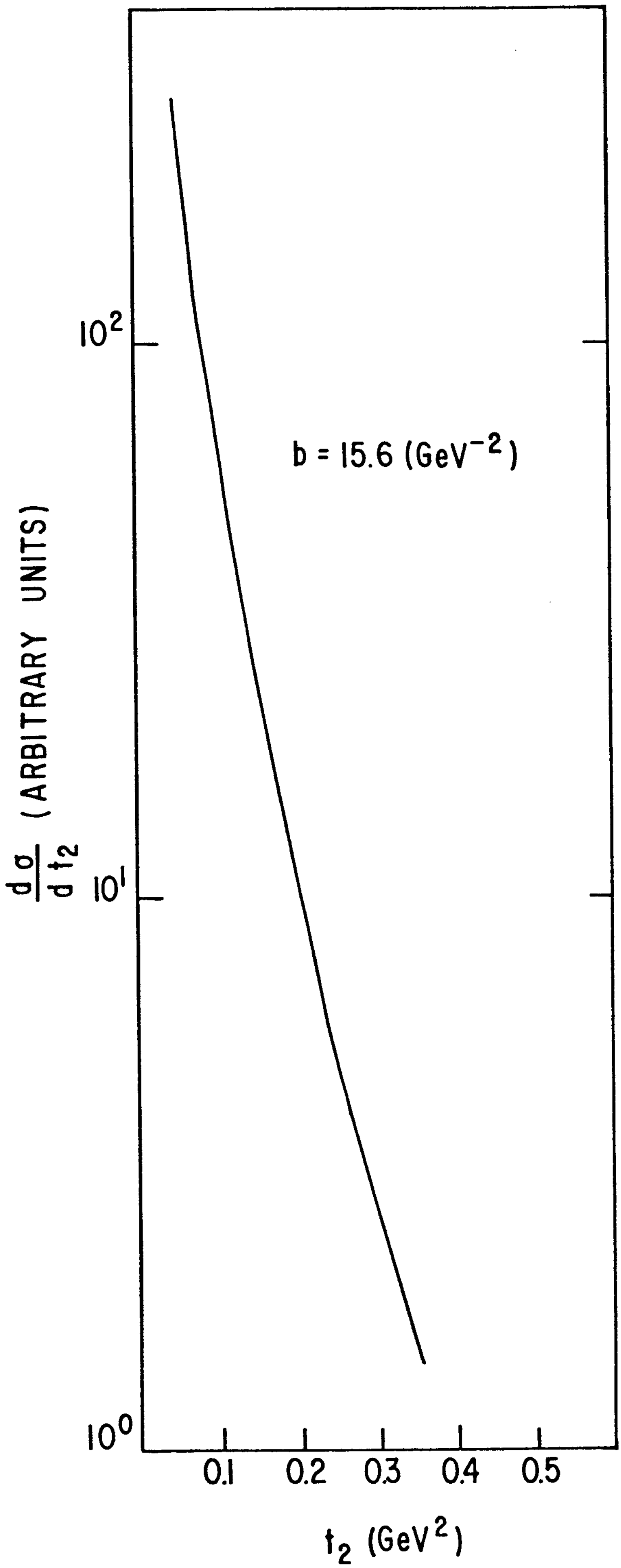
10^2
10^1
10^0
b = 15.6 (GeV^{-2})
$\frac{d\sigma}{dt_2}$ (ARBITRARY UNITS)
0.1
0.2
0.3
0.4
0.5
t_2 (GeV^2)

Fig. 6